Show that the following algorithm is valid for generating $X \sim geom(p)$:

1. Let i=0.
2. Generate U~U(0,1) independent of any previously generated U(0,1) random variates.
3. If $U \leq p$, return X = i. Otherwise, replace i by $i + 1$ and go back to step 2.

Note:
U(0,1) represents the uniform distribution where $0 \leq u \leq 1$.

Geom(p) represents the geometric distribution.
$f(x) = p(1-p)^x$, $x = 0,1,2\ldots$ is the pdf of the geometric distribution with mean $(1-p)/p$.

Please show all work, thanks.