8. Use the general solution (9) to solve the problem

$$
\begin{aligned}
c^2 y_{xx} &= y_{tt}, \qquad (0 < x < \infty,\ 0 < t < \infty) \\
y(x,0) = y_t(x,0) &= 0, \qquad (0 < x < \infty) \\
y(0,t) &= h(t), \qquad (0 < t < \infty)
\end{aligned}
$$

where $h(t)$ is prescribed. [We can imagine taking the left end of the string between two fingers and, beginning at $t = 0$, "jiggling" it according to $y(0,t) = h(t)$.]

(The problem is from Vibrating String; d'Alembert's Solution in Wave Equation.)
(Please give your complete solution and take a look at the answer for 8 below in order to check your final solution.)

The general solution (9):

$$\boxed{y(x,t) = F(x - ct) + G(x + ct),} \qquad (9)$$

The answer for 8:

$P(x) = p(x) + Kx^2/(2c^2)$. **8.** $y(x,t) = H(t - \frac{x}{c})h(t - \frac{x}{c})$. NOTE: $y(x,t) = F(x-ct) + G(x+ct)$ gives $y(x,0) = 0 = F(x) + G(x)$ and $y_t(x,0) = 0 = -cF'(x) + cG'(x)$. Solving these gives $F(x)$ = constant $\equiv A$, say, and $G(x) = -A$, so $y(x,t) = A - A = 0$. The key step is to realize that the two boundary condition equations hold only for $x > 0$, so $F(x) = 0$ (we can let $A = 0$ without loss) and $G(x) = 0$ hold only for $x > 0$. Thus, $F(x - ct) = 0$ for $x > ct$ (the wedge $0 < \theta < \pi/4$ in the first quadrant of the x, t plane) and $G(x + ct) = 0$ for $x > -ct$ (the wedge $0 < \theta < 3\pi/4$), so $y(x,t) = 0$ in $0 < \theta < \pi/4$ and $y(x,y) = F(x - ct)$ in $\pi/4 < \theta < \pi/2$. Then the boundary condition $y(0,t) = h(t) = F(-ct)$ gives $F(arg) = h\left(\frac{arg}{-c}\right)$, where "arg" denotes the argument of F, so $y(x,t) = F(x - ct) = h\left(\frac{x-ct}{-c}\right)$ holds in the wedge $\pi/4 < \theta < \pi/2$.