3. Textbook, Section 6.1, **12**.

12. Let $\beta = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be a basis for a vector space V, and let P be the mapping $P(a_1\mathbf{x}_1 + \cdots + a_n\mathbf{x}_n) = a_1\mathbf{x}_1 + \cdots + a_k\mathbf{x}_k$.

a) Show that $\mathrm{Ker}(P) = \mathrm{Span}(\{\mathbf{x}_{k+1}, \ldots, \mathbf{x}_n\})$ and $\mathrm{Im}(P) = \mathrm{Span}(\{\mathbf{x}_1, \ldots, \mathbf{x}_k\})$.

b) Show that $P^2 = P$.

c) Show conversely that if $P: V \rightarrow V$ is any linear mapping such that $P^2 = P$, then there exists a basis β for V such that P takes the form given in part a. (*Hint:* Show that $P^2 = P$ implies that $V = \mathrm{Ker}(P) \oplus \mathrm{Im}(P)$. These mappings are called *projections*. The orthogonal projections we studied in Chapter 4 are special cases.)