plots

Question 1 (20 marks)

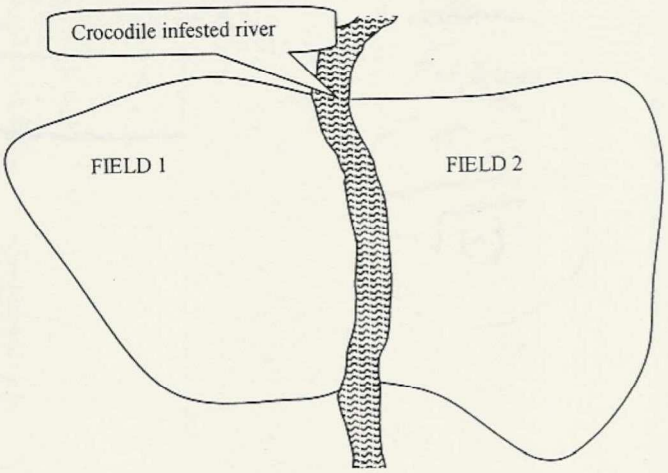
A game reserve has two grass fields separated by a crocodile infested river. Buck live on both sides of the river. As the ratio of buck to amount of grass decreases, buck try to cross the river if there is a better ratio on the other side but if they do, some get eaten by the crocodiles!

Variable	Description	units
b	Buck birth rate	[buck per buck per year]
d	Buck death rate	[buck per buck per year]
g	Grass growth rate	[kg per m^2 per year]
f	Rate at which buck eat grass	[kg per buck per year]
A_1, A_2	Area of fields 1 and 2	[m^2]
$r_1(x)$, $r_2(x)$	Rate of buck leaving field 1 or 2 respectively (Always positive – **see below**)	[buck per year]
$r_c(x)$	Rate of crocodiles eating buck – **see below**	[buck per year]
K	Rate constant for r_1 and r_2	[kg per year]

The rate of leaving field 1 or 2 is given by $\alpha = k\left(\frac{\text{buck in field 1}}{\text{kg grass in field 1}} - \frac{\text{buck in field 2}}{\text{kg grass in field 2}}\right)$

$$r_1(x) = \begin{cases} a & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases} \qquad r_2(x) = \begin{cases} -a & \alpha \leq 0 \\ 0 & \alpha > 0 \end{cases}.$$

The rate that crocodiles eat buck is given by $r_c(x) = 0.1 \times k|\alpha|$



Define the conserved quantities and write down the state space model of this system with output the total number of buck in the game reserve. Take care that the units in your model make sense.