Sheet 1

8. Working from first principles show that the condition for irrotationality of a two-dimensional ideal fluid flow is given by

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}.$$

[4 marks]

Hence define in mathematical terms the velocity potential ϕ,

[4 marks]

and show that potential lines of constant ϕ are perpendicular to streamlines of constant streamfunction ψ. You may assume that the gradient of the tangent to a streamline is given by $\mathrm{d}y/\mathrm{d}x = v/u$.

[4 marks]

By deriving the condition for continuity for the flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

show that the velocity potential ϕ satisfies Laplace's equation.

[4 marks]

Show that the flow given by $\phi = x^2 - y^2$ satisfies Laplaces equation.

[4 marks]