$X_1, ..., X_n$ are independent observations from a normal distribution with mean μ and variance σ^2. It can be shown that the sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 ,$$

where $\overline{X} = n^{-1}\sum_{i-1}^{n} X_i$, is distributed as $\Gamma(\alpha, \lambda)$ with $\alpha = (n-1)/2$ and $\lambda = (n-1)/2\sigma^2$.

a) Using the results for the gamma distribution, show that s^2 is an unbiased estimator of σ^2. Also state its variance.

b) Consider a statistic of the form $T_k = ks^2$, for some constant k. Find expressions for the bias and variance of T_k as an estimator of σ^2. Hence, or otherwise, find the value of k that minimises the mean squared error of T_k as an estimator of σ^2.