

(1) Let

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix}$$

Find a Jordan canonical form of A and a Jordan canonical basis for the linear transformation L_A associated to A

(2) Let

$$A = \begin{bmatrix} 2 & -4 & 2 & 2 \\ -2 & 0 & 1 & 3 \\ -2 & -2 & 3 & 3 \\ -2 & -6 & 3 & 7 \end{bmatrix}$$

Find the Jordan canonical form J for A and a matrix P such that $J = P^{-1}AP$.

(3) Let $V = P_2(\mathbb{R}) \subset \mathbb{R}[x, y]$ denote the vector space of polynomial functions with coefficients in \mathbb{R} in two variables of degree at most 2. A basis for V is $\alpha = \{1, x, y, x^2, y^2, xy\}$. Consider the linear mapping $T : V \rightarrow V$ defined by

$$T(f) = \frac{\partial}{\partial x} f$$

Find a Jordan canonical basis for T .