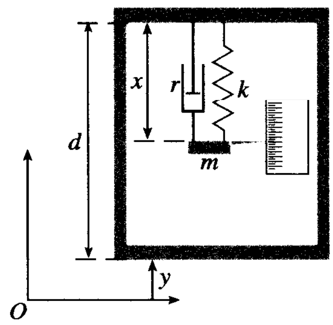
A seismograph is a scientific instrument that is used to detect earthquakes. A simple model of a seismograph is shown below. It consists of a particle of mass m to which a pointer is attached. The particle is suspended by a spring of natural length l_0 and stiffness k and a damper of damping constant r from a platform of height d which is fixed to the Earth. Let the vertical displacements of the Earth, relative to a fixed origin O, be denoted by y and let the length of the spring and the damper be x, as shown in the following diagram.



the displacement $x(t)$ of the pointer with respect to the platform satisfies the differential equation

$$m\ddot{x} + r\dot{x} + kx = mg + kl_0 + m\ddot{y}.$$ [5]

(d) Calculate the equilibrium length x_{eq} of the spring. [2]

(*Hint*: For this part you can assume that there is no seismic activity, i.e. $y(t) = \dot{y}(t) = 0$.)

(e) Define z to be the displacement of the pointer from its equilibrium position, i.e. $z = x - x_{eq}$. Find the differential equation satisfied by z. [2]

(f) The stiffness of the spring is $k = 80.48\,\text{N}\,\text{m}^{-1}$, and the mass of the particle is $0.8\,\text{kg}$. Find the natural frequency of the system. [2]

(g) What value of the damping constant r should be chosen in order to maximize the response of the equipment to a sinusoidal wave of frequency $10\,\text{Hz}$? [3]

(h) With the value of the damping constant chosen in part (g), find the magnification factors when the earthquake waves are of frequency:

(i) $8\,\text{Hz}$;

(ii) $10\,\text{Hz}$;

(iii) $12\,\text{Hz}$. [4]