

PROBLEMS

1. Solve the van der Waals equation for p as a function of V and T .

2. Given that $pV + a/V = RT$ and $U = (3/2)RT - a/V$, where a is constant. Find (a) $(\partial U/\partial T)_V$ and (b) $(\partial U/\partial T)_p$.

3. (a) Show that the coefficient of expansion $\beta = 1/T$ for an ideal gas. (b) Show that for a gas obeying the van der Waals equation

$$\beta = \frac{RV^2(V-b)}{RTV^3 - 2a(V-b)^2}$$

4. (a) Show that the compressibility $k = 1/p$ for an ideal gas. (b) Show that for a gas obeying the van der Waals equation

$$k = \frac{V^2(V-b)^2}{RTV^3 - 2a(V-b)^2}$$

5. If a wire expands on being heated, show that if a tension is applied adiabatically the temperature falls.

6. Find T_i , the Kelvin temperature of melting ice, when $p = 1$ atm, given that an increase in pressure of 1 atm depresses the melting point 0.0073°C , $V_1 = 0.0196$ m³/kg-mole for ice, $V_2 = 0.0180$ m³/kg-mole for water, and $L_f = 1.44 \times 10^6$ cal/kg-mole.

7. If the electromotive force for a certain cell is found to be given by $\mathcal{E} = 1.0000 + 0.00015(t - 15)$, where t is the temperature in $^\circ\text{C}$, find the heat of reaction at 25°C .

8. An ideal gas undergoes the following processes: (a) $V = k$, (b) $pV = k$, (c) $pV^2 = k$, (d) $p/V = k$, (e) $pT = k$, where k is a constant. If the initial state is given by $P_a = 1$ atm, $V_a = 24.6$ m³, $T_a = 300^\circ\text{K}$, and if in the final state $p_b = 2$ atm, find V_b and T_b for each process.

9. An ideal gas is taken around a Carnot cycle (see Fig. 2-1). At the temperature T_2 the volume increases from V_a to V_b , then the volume increases adiabatically from V_b to V_c , decreases isothermally (at temperature T_1) to V_d , and decreases adiabatically back to V_a . Find the work done by the gas and the heat taken in along each step, in terms of the V 's and T 's. Show that the efficiency is $(T_2 - T_1)/T_2$.

10. Prove that, in general, (a) $(\partial T/\partial V)_S = (C_V - C_p)/\beta C_V$, (b) $(\partial T/\partial p)_S = k(C_p - C_V)/\beta C_p$, and (c) $(\partial p/\partial V)_S = \gamma/kV$.

11. Taking C_V as constant, show that (a) $U_b - U_a = C_V(T_b - T_a)$ for an ideal gas, and (b) $U_b - U_a = C_V(T_b - T_a) + a/V_a - a/V_b$ for a van der Waals gas.

12. Show that for a van der Waals gas $S_b - S_a = C_V \ln(T_b/T_a) + R \ln(V_b - b)/(V_a - b)$.

13. Show that for a van der Waals gas

$$C_p - C_V = R \left[1 - \frac{2a(V-b)^2}{RTV^3} \right]^{-1}$$

14. (a) Suppose that with a certain gas in the porous-plug experiment it is observed that $C_p(\partial T/\partial p)_H = -k$, where k is a positive constant. Find the relation between V and T for such a gas. (b) Compare the behavior of such a gas with that of one whose equation of state is $p(V-b) = RT$.

15. Compute $T(\partial V/\partial T)_p - V$ for a van der Waals gas. (Neglect second-order small terms containing a^2 , ab , or b^2 , and for first-order small terms containing a or b consider $pV = RT$ to be a good enough approximation.) Show that the inversion temperature (below which the gas cools and above