A particle of mass $m$ moves in two dimensions under the influence of the potential $V\left(x,y\right)=\frac{1}{2}mω^{2}\left(6x^{2}-2xy+6y^{2}\right).$ Using the rotated coordinates $u=(x+y)/\sqrt{2}$ and $w=(x-y)/\sqrt{2}$ show that the Schrödinger equation in the new coordinates (u,w) is
$$-\frac{ℏ^{2}}{2m}\left(\frac{d^{2}}{du^{2}}+\frac{d^{2}}{dw^{2}}\right)ψ\left(u,w\right)+\overbar{V}\left(u,w\right)ψ\left(u,w\right)=Eψ\left(u,w\right)$$

Where $\overbar{V}(u,w)$ should be found.

Let$ ψ\left(u,w\right)=U\left(u\right)W(w)$. Use the technique of separation of variables to show that $U(u)$ and $W(w)$ satisfy the Schrödinger equations for the one dimensional quantum harmonic oscillator. Construct the allowed energy levels $E\_{n,m}$ and write down the corresponding wavefunction $ψ\_{m,n}(x,y)$.