Homework 2

Adapted from Fundamentals of Futures and Options Markets, 7th ed., John C. Hull.

Chapter 4

The 6-month, 12-month, 18-month, and 24-month zero rates are 3.00%, 3.50%, 4.00%, and 4.50% with semi-annual compounding.

Q1: What are the rates with continuous compounding?

Q2: What is the forward rate for the six-month period beginning in 12 months (i.e., $F_{12,18}$)?

Q3: What is the value of an FRA that promises to pay you 5.50% (compounded semi-annually) on a principal of $1 million for the six-month period starting in 12 months? Hints: you must convert your continuous rate calculated in Q2 to semi-annual to determine the current forward rate for the FRA; you must remember that the contract covers only 6 months (one-half year) of interest. See pages 90-93.

Business Snapshot 4.2 Orange County's yield curve plays

Suppose a large investor can borrow or lend at the rates in Table 4.5 and thinks that one-year interest rates will not change much over the next five years. The investor can borrow one-year funds and invest for five years. The one-year borrowings can be rolled over for further one-year periods at the end of the first, second, third, and fourth years. If interest rates do stay about the same, this strategy will yield a profit of about 2.3% per year because interest will be received at 5.3% and paid at 3%. This type of trading strategy is known as a *yield curve play*. The investor is speculating that rates in the future will be quite different from the forward rates observed in the market today. (In our example forward rates observed in the market today for future one-year periods are 5%, 5.8%, 6.2%, and 6.5%.)

Robert Citron, the Treasurer at Orange County, used yield curve plays similar to the one we have just described very successfully in 1992 and 1993. The profit from Mr. Citron's trades became an important contributor to Orange County's budget and he was re-elected. (No one listened to his opponent in the election, who said his trading strategy was too risky.)

In 1994 Mr. Citron expanded his yield curve plays. He invested heavily in *inverse floaters*. These pay a rate of interest equal to a fixed rate of interest minus a floating rate. He also leveraged his position by borrowing in the repo market. If short-term interest rates had remained the same or declined he would have continued to do well. As it happened, interest rates rose sharply during 1994. On December 1, 1994, Orange County announced that its investment portfolio had lost $1.5 billion and several days later it filed for bankruptcy protection.

fourth year. Because $122.14 = 114.80e^{0.062}$, money is being borrowed for the fourth year at the forward rate of 6.2%.

If a large investor thinks that rates in the future will be different from today's forward rates there are many trading strategies that the investor will find attractive (see Business Snapshot 4.2). One of these involves entering into a contract known as a *forward rate agreement*. We will now discuss how this contract works and how it is valued.

4.7 FORWARD RATE AGREEMENTS

A forward rate agreement (FRA) is an over-the-counter agreement that a certain interest rate will apply to either borrowing or lending a certain principal during a specified future period of time. The assumption underlying the contract is that the borrowing or lending would normally be done at LIBOR.

Consider a forward rate agreement where company X is agreeing to lend money at LIBOR to company Y for the period of time between T_1 and T_2. Define:

R_K: The rate of interest agreed to in the FRA

R_F: The forward LIBOR interest rate for the period between times T_1 and T_2, calculated today[3]

[3] LIBOR forward rates are calculated as described in Section 4.6 from the LIBOR/swap zero curve. The latter is determined in the way described in Section 7.6.

R_M: The actual LIBOR interest rate observed in the market at time T_1 for the period between times T_1 and T_2

L: The principal underlying the contract

We will depart from our usual assumption of continuous compounding and assume that the rates R_K, R_F, and R_M are all measured with a compounding frequency reflecting the length of the period they apply to. This means that, if $T_2 - T_1 = 0.5$, they are expressed with semiannual compounding; if $T_2 - T_1 = 0.25$, they are expressed with quarterly compounding; and so on. (This assumption corresponds to the usual market practices for FRAs.)

Normally company X would earn R_M from the LIBOR loan. The FRA means that it will earn R_K. The extra interest rate (which may be negative) that it earns as a result of entering into the FRA is $R_K - R_M$. The interest rate is set at time T_1 and paid at time T_2. The extra interest rate therefore leads to a cash flow to company X at time T_2 of

$$L(R_K - R_M)(T_2 - T_1) \tag{4.7}$$

Similarly there is a cash flow to company Y at time T_2 of

$$L(R_M - R_K)(T_2 - T_1) \tag{4.8}$$

From equations (4.7) and (4.8) we see that there is another interpretation of the FRA. It is an agreement where company X will receive interest on the principal between T_1 and T_2 at the fixed rate of R_K and pay interest at the realized market rate of R_M. Company Y will pay interest on the principal between T_1 and T_2 at the fixed rate of R_K and receive interest at R_M. This interpretation will be important when we come to consider swaps in Chapter 7.

Usually FRAs are settled at time T_1 rather than T_2. The payoff must then be discounted from time T_2 to T_1. For company X the payoff at time T_1 is

$$\frac{L(R_K - R_M)(T_2 - T_1)}{1 + R_M(T_2 - T_1)}$$

Example 4.2 Cash flows from an FRA

Suppose that a company enters into an FRA that specifies it will receive a fixed rate of 4% on a principal of \$100 million for a three-month period starting in three years. If three-month LIBOR proves to be 4.5% for the three-month period the cash flow to the lender will be

$$100{,}000{,}000 \times (0.040 - 0.045) \times 0.25 = -\$125{,}000$$

at the 3.25-year point. This is equivalent to a cash flow of

$$-\frac{125{,}000}{1 + 0.045 \times 0.25} = -\$123{,}609$$

at the three-year point. The cash flow to the party on the opposite side of the transaction will be +\$125,000 at the 3.25 point or +\$123,609 at the three-year point. (All interest rates in this example are expressed with quarterly compounding.)

and for company Y the payoff at time T_1 is

$$\frac{L(R_M - R_K)(T_2 - T_1)}{1 + R_M(T_2 - T_1)}$$

Example 4.2 illustrates the calculation of cash flows from an FRA.

Valuation

To value an FRA, we first note that it is always worth zero when $R_K = R_F$.[4] This is because, as noted in Section 4.6, a large financial institution can at no cost lock in the forward rate for a future time period. For example, it can ensure that it earns the forward rate for the time period between years 2 and 3 by borrowing a certain amount of money for two years and investing it for three years. Similarly, it can ensure that it pays the forward rate for the time period between years 2 and 3 by borrowing a certain amount of money three years and investing it for two years.

Compare two FRAs. The first promises that the LIBOR forward rate R_F will be earned on a principal of L between times T_1 and T_2; the second promises that R_K will be earned on the same principal between the same two dates. The two contracts are the same except for the interest payments received at time T_2. The excess of the value of the second contract over the first is, therefore, the present value of the difference between these interest payment, or

$$L(R_K - R_F)(T_2 - T_1)e^{-R_2T_2}$$

where R_2 is the continuously compounded riskless zero rate for a maturity T_2.[5] Because the value of the first FRA (where R_F is received) is zero, the value of the second FRA (where R_K is received) is

$$V_{\text{FRA}} = L(R_K - R_F)(T_2 - T_1)e^{-R_2T_2} \quad \textbf{(4.9)}$$

Similarly, the value of an FRA which promises that an interest rate of R_K will be paid on borrowings of L between T_1 and T_2 is

$$V_{\text{FRA}} = L(R_F - R_K)(T_2 - T_1)e^{-R_2T_2} \quad \textbf{(4.10)}$$

By comparing equations (4.7) and (4.9) (or equations (4.8) and (4.10)), we see that an

Example 4.3 Valuation of an FRA

Suppose that LIBOR zero and forward rates are as in Table 4.5. Consider an FRA where a company will receive a rate of 6%, measured with annual compounding, on a principal of \$100 million between the end of year 1 and the end of year 2. In this case, the forward rate is 5% with continuous compounding or 5.127% with annual compounding. From equation (4.9), it follows that the value of the FRA is

$$100{,}000{,}000 \times (0.06 - 0.05127)e^{-0.04\times 2} = \$805{,}800$$

[4] It is usually the case that R_K is set equal to R_F when the FRA is first initiated.

[5] Note that R_K, R_M, and R_F are expressed with a compounding frequency corresponding to $T_2 - T_1$, whereas R_2 is expressed with continuous compounding.

FRA can be valued if we:

1. Calculate the payoff on the assumption that forward rates are realized (that is, on the assumption that $R_M = R_F$)
2. Discount this payoff at the risk-free rate

Example 4.3 illustrates the valuation of FRAs.

4.8 THEORIES OF THE TERM STRUCTURE OF INTEREST RATES

It is natural to ask what determines the shape of the zero curve. Why is it sometimes downward sloping, sometimes upward sloping, and sometimes partly upward sloping and partly downward sloping? A number of different theories have been proposed. The simplest is *expectations theory*, which conjectures that long-term interest rates should reflect expected future short-term interest rates. More precisely, it argues that a forward interest rate corresponding to a certain future period is equal to the expected future zero interest rate for that period. Another idea, *market segmentation theory*, conjectures that there need be no relationship between short-, medium-, and long-term interest rates. Under the theory, a major investor such as a large pension fund invests in bonds of a certain maturity and does not readily switch from one maturity to another. The short-term interest rate is determined by supply and demand in the short-term bond market; the medium-term interest rate is determined by supply and demand in the medium-term bond market; and so on.

The theory that is most appealing is *liquidity preference theory*. The basic assumption underlying the theory is that investors prefer to preserve their liquidity and invest funds for short periods of time. Borrowers, on the other hand, usually prefer to borrow at fixed rates for long periods of time. This leads to a situation in which forward rates are greater than expected future zero rates. The theory is also consistent with the empirical result that yield curves tend to be upward sloping more often than they are downward sloping.

The Management of Net Interest Income

To understand liquidity preference theory, it is useful to consider the interest rate risk faced by banks when they take deposits and make loans. The *net interest income* of the bank is the excess of the interest received over the interest paid and needs to be carefully managed.

Consider a simple situation where a bank offers consumers a one-year and a five-year deposit rate as well as a one-year and five-year mortgage rate. The rates are shown in Table 4.6. We make the simplifying assumption that the expected one-year interest rate for future time periods to equal the one-year rates prevailing in the market today. Loosely speaking this means that the market considers interest rate increases to be just as likely as interest rate decreases. As a result, the rates in Table 4.6 are "fair" in that they reflect the market's expectations (i.e., they correspond to expectations theory). Investing money for one year and reinvesting for four further one-year periods give the same expected overall return as a single five-year investment. Similarly, borrowing