1. Let f : [0, 2] → R be continuous, assume that f is twice diﬀerentiable at all points of (a, b), and assume that f(0) = 0, f(1) = 1 and f(2) = 2. Prove: There exists c ∈ (0, 2) such that f′′(c) = 0.

2. Let f : (a, b) → R, where a, b ∈ R and a < b and suppose f is monotone. Prove lim x→c+ f(x) and lim x→c− f(x) exist at all c ∈ (a, b).

3. Let f : [0, 1] → R be continuous, diﬀerentiable at all points of (0, 1). Assume f′(x) ≥ 16 for all x ∈ (0, 1). Prove there is some interval J ⊂ [0, 1] of length 1/4 such that |f(x)| ≥ 4 for all x ∈ J.

4. Let I be an open interval in R, f : I → R and suppose that f satisﬁes the following condition: There exist constants C and α, C > 0 and α > 1, such that |f(x) − f(y)| <C|x − y|αfor all x, y ∈ I. Prove: f is constant. (A function that satisﬁes a Holder condition of order α with α > 1 is a constant).