We have

$$u(\rho, \theta, z) = -z\rho^4$$

The partial derivatives of u are:

$$\frac{\partial u}{\partial \rho} = -4z\rho^3, \qquad \frac{\partial u}{\partial \theta} = 0, \qquad \frac{\partial u}{\partial z} = -\rho^4$$

So we can use Equation (4.7) - Unit 23, p30 - to obtain

$$\nabla u = \underline{\text{grad}}\, u = \underline{e}_\rho \frac{\partial u}{\partial \rho} + \underline{e}_\theta \frac{1}{\rho}\frac{\partial u}{\partial \theta} + \underline{e}_z \frac{\partial u}{\partial z}$$

$$= (-4z\rho^3)\underline{e}_\rho + (-\rho^4)\underline{e}_z$$

So,

$$-\nabla u = -\left(-4z\rho^3 \underline{e}_\rho + (-\rho^4)\underline{e}_z\right)$$

$$= (4z\rho^3)\underline{e}_\rho + (\rho^4)\underline{e}_z$$

where e_ρ, and $\underline{e}_z$ are unit vectors in the ρ-direction and z-direction.

Since $\underline{V} = -\underline{\text{grad}}\, u$ (from part (iv)), we have:

$$\underline{V} = -\underline{\text{grad}}\, u = 4z\rho^3 \underline{e}_\rho + \rho^4 \underline{e}_z$$