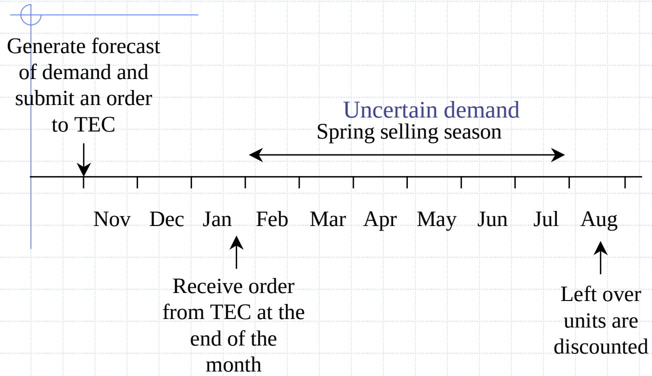
GMAN 505: Forecasting and Operations Management

Class 8:

Betting on Uncertain Demand

O'Neill's Hammer 3/2 wetsuit

Hammer 3/2 timeline and economics



Economics:

- Each suit sells for p = \$180
- TEC charges c = \$110 per suit
- Discounted suits sell for v = \$90

The "too much/too little problem":

Order too much and inventory is left over at the end of the season

Order too little and sales are lost.

Marketing's forecast for sales is 3200 units.

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Newsvendor model implementation steps

- Gather economic inputs:
  - Selling price, production/procurement cost, salvage value of inventory
- Generate a demand model:
  - Use empirical demand distribution or choose a standard distribution function to represent demand, e.g. the Normal distribution, the Poisson distribution.
- Choose an objective, e.g. maximize expected profit.
- Choose a quantity to order.

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The Critical Ratio

- C_o = overage cost - cost *per unit* of over ordering
  - For the Hammer 3/2 $C_o = c - v = 110 - 90 = 20$
- C_u = underage cost - cost *per unit* of under ordering
  - For the Hammer 3/2 $C_u = p - c = 180 - 110 = 70$
- The ratio $C_u / (C_o + C_u)$ is called the *critical ratio*.

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Expected profit maximizing order quantity

- $F(Q)$ = probability realized demand is Q or lower, i.e., $F(Q)$ is the distribution function
- With the profit maximizing order quantity, Q, the expected loss on the Q^{th} unit equals the expected gain on the Q^{th} unit

$$C_o \times F(Q) = C_u \times (1 - F(Q))$$

- Rearrange terms in the above equation ->

$$F(Q) = \frac{C_u}{C_o + C_u}$$

- Hence, we seek an order quantity Q such that the probability demand is less than Q equals the critical ratio. That Q maximizes expected profit.

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Normal distribution: density function

- The density function of a distribution provides the probability the outcome will precisely equal a value.

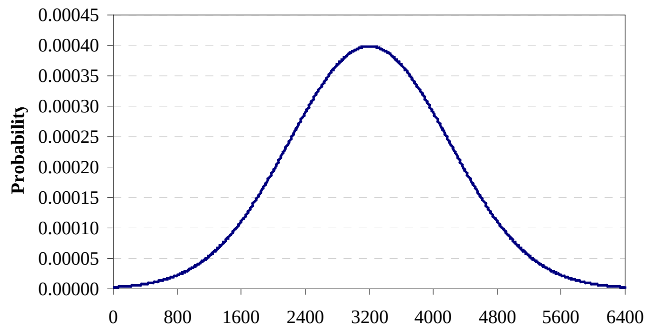

Figure 7.2: The density function, *f(Q)* , of a Normal distribution with mean=3200 and standard deviation=1000

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Normal distribution function

- The distribution function of a distribution gives the probability the outcome will be less than or equal to a value.
  - The Normal distribution is defined by its mean, μ, and its standard deviation, σ

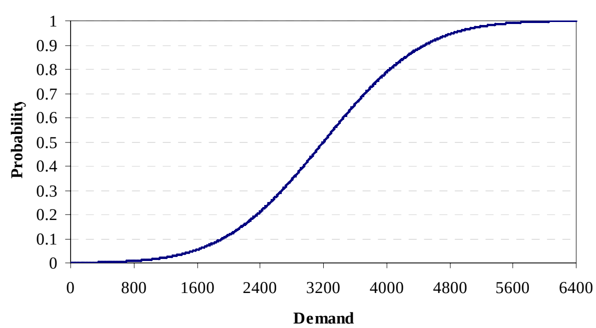

Figure 7.1: The distribution function, *F(Q)* , of a Normal distribution with mean=3200 and standard deviation=1000

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Normal distribution tutorial

- Suppose our demand model is a Normal distribution with μ = 3192 and σ = 1181.
- Examples of the questions we are interested in …
  - What is the probability demand is less than 3400?
  - With what order quantity is there a 85% probability that demand is less than that order quantity?
- Each of these questions is solved by relating our demand model to the Standard Normal distribution which has μ = 0 and σ = 1.

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Probability demand is less than Q

- Find the *z*-statistic that corresponds to Q : $z = (Q - \mu)/\sigma$
  - Lookup Φ(z) in the *Std Normal Distribution Function Table*.
  - Φ(z) is the probability demand will be less than *Q*.
- Example:
  - *z*-statistic for *Q* = 3400 is (3400-3192)/1181 = 0.18
  - A portion of the *Std Normal Distribution Function Table*:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141

- Answer: 57.14%
  - Alternative to using the table, use *Normsdist*(*z*) in Excel
- Probability demand is greater than *Q* = 1 - Φ(z) = 42.86%

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Normal distribution tutorial (cont)

- Find a *Q* such that there is a 85% probability demand is less than *Q*.
  - Find the *z*-statistic in the *Std Normal Dist. Function Table* such that $\Phi(z) = 0.85$:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830

  - $\Phi(1.03) = 0.8485$ and $\Phi(1.04) = 0.8508$, so let's choose $z = 1.04$
  - Now convert the chosen z-statistic into a *Q*:

$$Q = \mu + z \times \sigma = 3192 + 1.04 \times 1181 = 4420$$

  - Q = 4420 is also the quantity such that there is a 1-0.85 = 0.15 probability demand exceeds that quantity.

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Expected profit maximizing order quantity using the Normal distribution

- Inputs, underage cost, overage cost and critical ratio (0.7778) are the same as evaluated before.
- Lookup *z*-statistic:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

  - If the critical ratio falls between two values in the table, choose the greater *z*-statistic
  - Choose $z = 0.77$
- Convert the z-statistic into an order quantity:

$$Q = \mu + z \times \sigma$$
$$= 3192 + 0.77 \times 1181 = 4101$$

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Newsvendor model summary

- The model can be applied to settings in which …
  - There is a single order//replenishment opportunity.
  - Demand is uncertain.
  - If demand exceeds the order quantity, sales are lost.
  - If demand is less than the order quantity, there is left over inventory.
- Firm must have a demand model that includes an expected demand and standard deviation.
- At the order quantity that maximizes expected profit the probability that demand is less than the order quantity equals the critical ratio:
  - The expected profit maximizing order quantity balances the “too much-too little” costs.

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LL Bean order quantities

- Problem description:
  - Single item with a single ordering opportunity and one selling season with random demand.
  - Selling price = p = $45.
  - Cost = c = $25
  - End of season discount price = v = $15
  - Demand forecast = 12000 units
- Initial analysis:
  - Underage cost = $C_u = p - c = 45 - 25 = 20$
  - Overage cost = $C_o = c - v = 25 - 15 = 10$
  - Critical ratio = $C_u / (C_o + C_u) = 20 / 30 = 0.6667$

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Pick Q with a Normal distribution

- Optimal Q when demand is represented by a Normal distribution with mean 13,080 and standard deviation 4,320:
  - The z-statistic for a critical ratio of 0.6667 is 0.44: $\Phi(0.43) = 0.6664$ and $\Phi(0.44) = 0.6700$
  - $Q = \mu + z \times \sigma = 13{,}080 + 0.44 \times 4320 = 14{,}981$

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Newsvendor model performance measures

- *Expected lost sales*
  - The average number of units demand exceeds the order quantity
- *Expected sales*
  - The average number of units sold.
- *Expected left over inventory*
  - The average number of units left over at the end of the season.
- *Expected profit*
- *Expected gross margin percentage*
  - The gross margin (revenue – cost) as a percentage of revenue
- *Expected fill rate*
  - The fraction of demand that is satisfied immediately
- *In-stock/stockout probability*
  - Probability all demand is satisfied
- *Stockout probability*
  - Probability some demand is lost

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Expected lost sales with 3500 Hammer 3/2s

- Definition:
  - if demand is 3800 and Q = 3500, then lost sales is 300
  - If demand is 3200 and Q = 3500, then lost sales is 0
  - Expected lost sales is the average over all possible demand outcomes.
- If demand is Normally distributed:
  - Step 1: normalize the order quantity to find its z-statistic.

$$z = \frac{Q-\mu}{\sigma} = \frac{3500-3192}{1181} = 0.26$$

  - Step 2: Lookup in the *Std Normal Loss Function Table* the expected lost sales for a Std Normal distribution with that z-statistic: $L(0.26)=0.2824$
    - or, in Excel $L(z) = Normdist(z,0,1,0) - z*(1 - Normsdist(z))$
  - Step 3: Evaluate lost sales for the actual Normal distribution:

$$Expected\ lost\ sales = \sigma \times L(z) = 1181 \times 0.2824 = 333$$

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Measures that follow expected lost sales

Expected sales = μ - *Expected lost sales* = 3192 – 333 = 2859

Expected Left Over Inventory = Q - *Expected Sales* = 3500 – 2859 = 641

Expected Profit= C_u* *Expected Sales* - C_o**Expected Left Over Inventory*

Revenue= *p* Expected Sales +v*Expected Left Over Inventory*

Cost =*c*Q=11*3500=385,000*

Expected Fill Rate = *Expected Sales/Expected Demand= Expected Sales/* μ *= 2859/3192=89.6%*

Note: the above equations hold for any demand distribution

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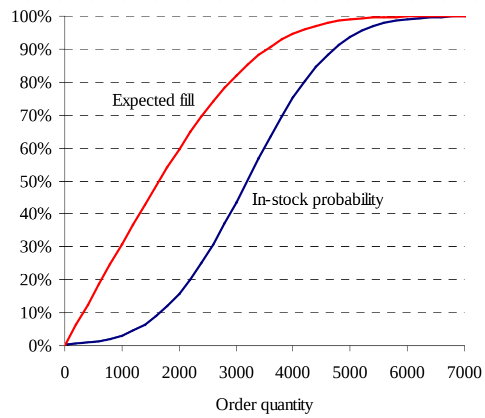
Service measures of performance

In-stock probability = $F(Q) = \Phi(z)$

the z-statistic for the order quantity: $z = \frac{Q-\mu}{\sigma} = \frac{3500-3192}{1181} = 0.26$

Lookup $\Phi(z)$ in the *Std. Normal Distribution Function Table*, $\Phi(0.26)$=60.29%



Stockout probability = 1 – $F(Q)$ = =1 – *In-stock probability* = 1 –0.6029 = 39.71%

Choose Q subject to a minimum in-stock probability

- Suppose we wish to find the order quantity for the Hammer 3/2 that minimizes left over inventory while generating at least a 99% in-stock probability.
- Step 1:
  - Find the z-statistic that yields the target in-stock probability.
  - In the *Std. Normal Distribution Function Table* we find $\Phi(2.32) = 0.9898$ and $\Phi(2.33) = 0.9901$.
  - Choose $z = 2.33$ to satisfy our in-stock probability constraint.
- Step 2:
  - Convert the z-statistic into an order quantity for the actual demand distribution.
  - $Q = \mu + z \times \sigma = 3192 + 2.33 \times 1181 = 5944$

Choose Q subject to a minimum fill rate constraint

- Suppose we wish to find the order quantity for the Hammer 3/2 that minimizes left over inventory while generating at least a 99% fill rate.
- Step 1:Find the lost sales with a Std Normal distribution that yields the target fill rate.

$$L(z)=\left(\frac{\mu}{\sigma}\right)(1\text{-}\ Fill\ rate)=\left(\frac{3192}{1181}\right)(1-0.99)=0.0270$$

- Step 2: Find the z-statistic that yields the lost sales found in step 1.
  - From the *Std Normal Loss Function Table*, $L(1.53)=0.0274$ and $L(1.54) = 0.0267$
  - Choose the higher z-statistic, $z = 1.54$
- Step 3: Convert the z-statistic into an order quantity for the actual demand distribution.
  - $Q = \mu + z \times \sigma = 3192 + 1.54 \times 1181 = 5011$

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