1. Write out the chain rule for each of the following functions and justify your answer in each case using this following Theorem.

Theorem: Chain Rule

Let $U ⊂ R^{m}⟶R^{p}$ and $f:V⊂ R^{m}⟶R^{p}$ be given functions such that $g$ maps $U$ into $V$, so that $f∘g$ is defined. Suppose $g$ is differentiable at $X\_{0}$ and $f$ is differentiable at $y\_{0}=g(X\_{0})$. Then $f∘g$ is differentiable at $X\_{0}$ and $D\left(f∘g\right)\left( X\_{0}\right)=Df\left(y\_{0}\right)Dg(X\_{0})$. The right-hand side is the matrix product of $Df\left(y\_{0}\right)$ with $Dg(X\_{0})$.

1. $\frac{δh}{δx}$ where $h\left(x,y\right)=f\left(x,u\left(x,y\right)\right)$
2. $\frac{dh}{dx}$ where $h\left(x\right)=f\left(x,u\left(x\right),v\left(x\right)\right)$
3. $\frac{δh}{δx}$ where $h\left(x,y,z\right)=f\left(u\left(x,y,z\right), v\left(x,y\right), w\left(x\right)\right)$
4. Verify the chain rule for $\frac{δh}{δx}$, where $h\left(x,y\right)=f(u\left(x,y\right), v\left(x,y\right))$ and

$f\left(u,v\right)=\frac{u^{2}+v^{2}}{u^{2}-v^{2}}$ , $ u\left(x,y\right)=e^{-x-y}$ , $v\left(x,y\right)=e^{xy}$.

1. Suppose that the temperature at the point $(x,y,z)$ in space is $T\left(x,y,z\right)=x^{2}+y^{2}+z^{2}$. Let a particle follow the right-circular helix $σ\left(t\right)=(\cos(t,\sin(t, t)))$ and let $T(t)$ be its temperature at time $t$. What is $T^{'}(t)$?
2. Captain Ralph is in trouble near the sunny side of Mercury. The temperature of the ship’s hull when he is at location $(x,y,z)$ will be given by $T\left(x,y,z\right)=e^{-x^{2}-2y^{2}-3z^{2}}$, where $x,y,$ and $z$ are measured in meters. He is currently at $\left(1,1,1\right).$
3. In what direction should he proceed in order to decrease the temperature most rapidly?
4. If the ship travels at $e^{8}$ meters per second, how fast will be the temperature decrease if the proceeds in that direction?
5. Unfortunately, the metal of the hull will crack if cooled at a rate greater than $\sqrt{14}e^{2}$ degrees per second. Describe the set of possible directions in which he may proceed to bring the temperature down at no more than that rate.