4. Let $\mathbf{u}, \mathbf{v} \in \mathbb{C}^n$ and set $A := I + \mathbf{u}\mathbf{v}^H \in \mathbb{C}^{n \times n}$.

(a) Suppose A is invertible. Prove that $A^{-1} = I_n + \alpha \mathbf{u}\mathbf{v}^H$, for some $\alpha \in \mathbb{C}$. Give an expression for α.

(b) For what $\mathbf{u}$ and $\mathbf{v}$ is A singular?

(c) Suppose A is singular. What is the null space of A, $N(A)$ in this case?