Exercises 97–100: (Refer to the discussion about visualizing exponents found in this section.) Make a sketch of blocks that represents the given exponential equation.

97. $4^2 \cdot 4^1 = 4^3$

98. $2^3 \cdot 2^3 = 2^6$

99. $2^3 \cdot 3^2 \cdot 3^1 \cdot 2^2 = 2^5 \cdot 3^3$

100. $4^2 \cdot 2^2 \cdot 4^2 = 4^5$ (*Hint:* Write 2^2 with a different base.)

WRITING ABOUT MATHEMATICS

101. Are the expressions $(4x)^2$ and $4x^2$ equal? Explain your answer.

102. Are the expressions $3^3 \cdot 2^3$ and 6^6 equal? Explain your answer.

5.2 ADDITION AND SUBTRACTION OF POLYNOMIALS

Monomials and Polynomials ▪ Addition of Polynomials ▪ Subtraction of Polynomials ▪ Evaluating Polynomial Expressions

A LOOK INTO MATH ▷

If you have ever exercised strenuously and then taken your pulse immediately afterward, you may have discovered that your pulse slowed quickly at first and then gradually leveled off. A typical scatterplot of this phenomenon is shown in Figure 5.4(a). These data points cannot be modeled accurately with a line, so a new expression, called a *polynomial,* is needed to model them. A graph of this new expression is shown in Figure 5.4(b) and discussed in Exercise 79. (Source: V. Thomas, *Science and Sport.*)

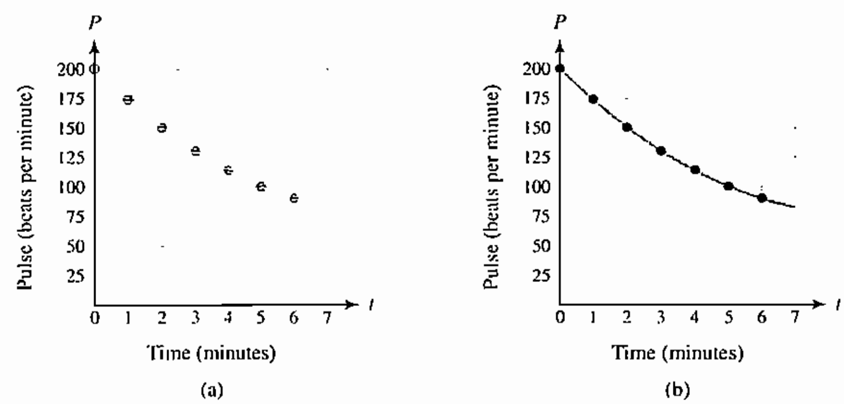


Figure 5.4 Heart Rate After Exercising

Monomials and Polynomials

A **monomial** is a number, a variable, or a product of numbers and variables raised to natural number powers. Examples of monomials include

$$-3, \quad xy^2, \quad 5a^2, \quad -z^3, \quad \text{and} \quad -\frac{1}{2}xy^3.$$

A monomial may contain more than one variable, but monomials do not contain division by variables. For example, the expression $\frac{3}{z}$ is not a monomial. If an expression contains addition or subtraction signs, it is *not* a monomial.

The **degree of a monomial** is the sum of the exponents of the variables. If the monomial has only one variable, its degree is the exponent of that variable. Remember, when a variable does not have a written exponent, the exponent is implied to be 1. A nonzero number has degree 0, and the number 0 has *undefined* degree. The number in a monomial is called the **coefficient of the monomial**. Table 5.2 contains the degree and coefficient of several monomials.

TABLE 5.2 Properties of Monomials

Monomial	-5	$6a^3b$	$-xy$	$7y^3$
Degree	0	4	2	3
Coefficient	-5	6	-1	7

A **polynomial** is the sum of one or more monomials. Each monomial is called a *term* of the polynomial. Addition or subtraction signs separate terms. The expression $2x^2 - 3x + 5$ is a **polynomial in one variable** with three terms. Examples of polynomials in one variable include

$$-2x, \quad 3x + 1, \quad 4y^2 - y + 7, \quad \text{and} \quad x^5 - 3x^3 + x - 7.$$

These polynomials have 1, 2, 3, and 4 terms, respectively. A polynomial with *two terms* is called a **binomial**, and a polynomial with *three terms* is called a **trinomial**.

A polynomial can have more than one variable, as in

$$x^2y^2, \quad 2xy^2 + 5x^2y - 1, \quad \text{and} \quad a^2 + 2ab + b^2.$$

Note that all variables in a polynomial are raised to natural number powers. The **degree of a polynomial** is the degree of the term (or monomial) with highest degree.

EXAMPLE **Identifying properties of polynomials**

Determine whether the expression is a polynomial. If it is, state how many terms and variables the polynomial contains and its degree.

(a) $7x^2 - 3x + 1$ **(b)** $5x^3 - 3x^2y^3 + xy^2 - 2y^3$ **(c)** $4x^2 + \dfrac{5}{x + 1}$

Solution

(a) The expression $7x^2 - 3x + 1$ is a polynomial with three terms and one variable. The first term $7x^2$ has degree 2 because the exponent on the variable is 2. The second term $-3x$ has degree 1 because the exponent on the variable is implied to be 1. The third term 1 has degree 0 because it is a nonzero number. The term with highest degree is $7x^2$, so the polynomial has degree **2**.

(b) The expression $5x^3 - 3x^2y^3 + xy^2 - 2y^3$ is a polynomial with four terms and two variables. The first term has degree 3 because the exponent on the variable is 3. The second term has degree 5 because the *sum* of the exponents on the variables is 5. Likewise, the third term has degree 3 and the fourth term has degree 3. The term with highest degree is $-3x^2y^3$, so the polynomial has degree $\mathbf{2 + 3} = 5$.

(c) The expression $4x^2 + \frac{5}{x + 1}$ is not a polynomial because it contains division by the polynomial $x + 1$.

Now Try Exercises

Addition of Polynomials

Suppose that we have 2 identical rectangles with length L and width W, as illustrated in Figure 5.5. Then the area of one rectangle is LW and the total area is

$$LW + LW.$$

This area is equivalent to 2 times LW, which can be expressed as $2LW$, or

$$LW + LW = 2LW.$$

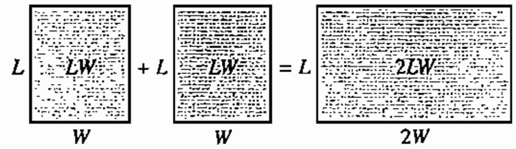


Figure 5.5 Adding $LW + LW$

If two monomials contain the same variables raised to the same powers, we call them **like terms**. We can add or subtract *like* terms but not *unlike* terms. The terms LW and $2LW$ are like terms and can be combined geometrically, as shown in Figure 5.6. If we joined one of the small rectangles with area LW and a larger rectangle with area $2LW$, then the total area is $3LW$.

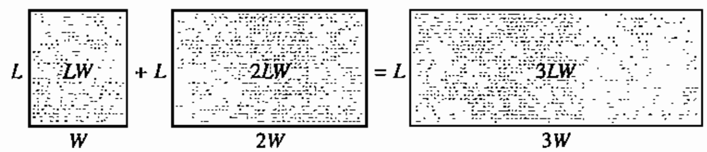


Figure 5.6 Adding $LW + 2LW$

The *distributive property* justifies combining like terms.

$$1LW + 2LW = (1 + 2)LW = 3LW$$

The rectangles shown in Figure 5.7 have areas of ab and xy. Their combined area is their sum, $ab + xy$. However, because these monomials are unlike terms, they cannot be combined into one term.

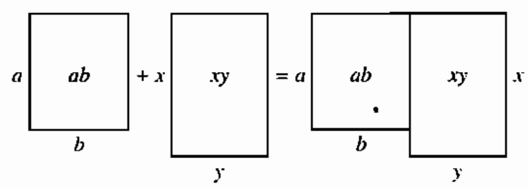


Figure 5.7 Unlike terms: $ab + xy$

EXAMPLE 2

Adding like terms

State whether each pair of expressions contains like terms or unlike terms. If they are like terms, add them.

(a) $5x^2, -x^2$ **(b)** $7a^2b, 10ab^2$ **(c)** $4rt^2, \frac{1}{2}rt^2$

Solution

(a) The terms $5x^2$ and $-x^2$ have the same variable raised to the same power, so they are like terms. To add like terms add their coefficients. Note that the coefficient of $-x^2$ is -1.

$$5x^2 + (-x^2) = (5 + (-1))x^2 \quad \text{Distributive property}$$
$$= 4x^2 \quad \text{Add.}$$

(b) The terms $7a^2b$ and $10ab^2$ have the same variables, but these variables are not raised to the same power. They are unlike terms and therefore cannot be added.

(c) The terms $4rt^2$ and $\frac{1}{2}rt^2$ have the same variables raised to the same powers, so they are like terms.

$$4rt^2 + \frac{1}{2}rt^2 = \left(4 + \frac{1}{2}\right)rt^2 \quad \text{Distributive property}$$

$$= \frac{9}{2}rt^2 \quad \text{Add.}$$

Now Try Exercises 33, 35, 39

To add two polynomials, add like terms, as illustrated in the next example.

EXAMPLE 3 Adding polynomials

Add each pair of polynomials by combining like terms.

(a) $(3x + 4) + (-4x + 2)$

(b) $(y^2 - 2y + 1) + (3y^2 + y + 11)$

(c) $(3a^3 - 4a + 1) + (4a^3 + a^2 - 5)$

Solution

(a) $(3x + 4) + (-4x + 2) = 3x + (-4x) + 4 + 2$

$= (3 - 4)x + (4 + 2)$

$= -x + 6$

(b) $(y^2 - 2y + 1) + (3y^2 + y + 11) = y^2 + 3y^2 - 2y + y + 1 + 11$

$= (1 + 3)y^2 + (-2 + 1)y + (1 + 11)$

$= 4y^2 - y + 12$

NOTE: With practice the first two steps can be done mentally.

(c) $(3a^3 - 4a + 1) + (4a^3 + a^2 - 5) = 3a^3 + 4a^3 + a^2 - 4a + 1 - 5$

$= (3 + 4)a^3 + a^2 - 4a - 4$

$= 7a^3 + a^2 - 4a - 4$

Now Try Exercises 41, 43, 47

Polynomials can also be added vertically, as demonstrated in the next example.

EXAMPLE 4 Adding polynomials vertically

Simplify $(3x^2 - 3x + 5) + (-x^2 + x - 6)$.

Solution

Write the polynomials in a vertical format and then add each column of like terms.

$$\begin{array}{r} 3x^2 - 3x + 5 \\ \underline{-x^2 + x - 6} \\ 2x^2 - 2x - 1 \end{array} \quad \text{Add.}$$

Regardless of the method used, the same answer should be obtained. However, adding vertically requires that *like terms be placed in the same column.* Now Try Exercise 53

Subtraction of Polynomials

To subtract one integer from another, add the first integer with the *additive inverse* or *opposite* of the second integer. For example, $3 - 5$ is evaluated as follows.

$$3 - 5 = 3 + (-5) \quad \text{Add the opposite.}$$
$$= -2 \quad \text{Simplify.}$$

Similarly, to subtract one polynomial from another, add the first polynomial and the *opposite* of the second polynomial. To find the opposite of a polynomial, simply negate each term. Table 5.3 lists some polynomials and their opposites.

CRITICAL THINKING

What is the result when a polynomial and its opposite are added?

TABLE 5.3 Opposites of Polynomials

Polynomial	Opposite
$2x - 4$	$-2x + 4$
$-x^2 - 2x + 9$	$x^2 + 2x - 9$
$6x^3 - 12$	$-6x^3 + 12$
$-3x^4 - 2x^2 - 8x + 3$	$3x^4 + 2x^2 + 8x - 3$

EXAMPLE 5 **Subtracting polynomials**

Simplify each expression.
(a) $(3x - 4) - (5x + 1)$
(b) $(5x^2 + 2x - 3) - (6x^2 - 7x + 9)$
(c) $(6x^3 + x^2) - (-3x^3 - 9)$

Solution

(a) To subtract $(5x + 1)$ from $(3x - 4)$, we add the opposite of $(5x + 1)$, or $(-5x - 1)$.

$$\begin{aligned}(3x - 4) - (5x + 1) &= (3x - 4) + (-5x - 1)\\ &= (3 - 5)x + (-4 - 1)\\ &= -2x - 5\end{aligned}$$

(b) The opposite of $(6x^2 - 7x + 9)$ is $(-6x^2 + 7x - 9)$.

$$\begin{aligned}(5x^2 + 2x - 3) - (6x^2 - 7x + 9) &= (5x^2 + 2x - 3) + (-6x^2 + 7x - 9)\\ &= (5 - 6)x^2 + (2 + 7)x + (-3 - 9)\\ &= -x^2 + 9x - 12\end{aligned}$$

(c) The opposite of $(-3x^3 - 9)$ is $(3x^3 + 9)$.

$$\begin{aligned}(6x^3 + x^2) - (-3x^3 - 9) &= (6x^3 + x^2) + (3x^3 + 9)\\ &= (6 + 3)x^3 + x^2 + 9\\ &= 9x^3 + x^2 + 9\end{aligned}$$

Now Try Exercises 63, 65, 69

NOTE: Some students prefer to subtract one polynomial from another by noting that a subtraction sign in front of parentheses changes the signs of all of the terms within the parentheses. For example, part (a) of the previous example could be worked as follows.

$$\begin{aligned}(3x - 4) - (5x + 1) &= 3x - 4 - 5x - 1 \\ &= (3 - 5)x + (-4 - 1) \\ &= -2x - 5\end{aligned}$$

EXAMPLE 6 **Subtracting polynomials vertically**

Simplify $(5x^2 - 2x + 7) - (-3x^2 + 3)$.

Solution

To subtract one polynomial from another vertically, simply add the first polynomial and the opposite of the second polynomial. No x-term occurs in the second polynomial, so insert $0x$.

$5x^2 - 2x + 7$	
$3x^2 + 0x - 3$	Opposite of $-3x^2 + 3$ is $3x^2 - 3$ or $3x^2 + 0x - 3$.
$8x^2 - 2x + 4$	Add like terms in each column.

Now Try Exercise 77

Evaluating Polynomial Expressions

Frequently, monomials and polynomials represent formulas that may be evaluated. We illustrate such applications in the next two examples.

EXAMPLE 7 **Writing and evaluating a monomial**

Write the monomial that represents the volume of the box having a square bottom, as shown in Figure 5.8. Find the volume of the box if $x = 3$ feet and $y = 2$ feet.

CRITICAL THINKING

Write an expression that gives the volume of six identical cubes having sides of length L.

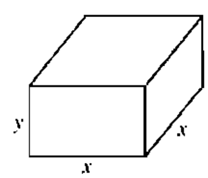


Figure 5.8

Solution

The volume of a box is found by multiplying the length, width, and height together. Because the length and width are both x and the height is y, the monomial xxy represents the volume of the box. This can be written x^2y. To calculate the volume let $x = 3$ and $y = 2$ in the monomial x^2y.

$$x^2y = 3^2 \cdot 2 = 9 \cdot 2 = 18 \text{ cubic feet}$$

Now Try Exercise 81

EXAMPLE 8 **Modeling sales of personal computers**

Worldwide sales of personal computers have increased dramatically in recent years, as illustrated in Figure 5.9. The polynomial

$$0.7868x^2 + 12x + 79.5$$

approximates the number of computers sold in millions, where $x = 0$ corresponds to 1997, $x = 1$ to 1998, and so on. Estimate the number of personal computers sold in 2002 by using both the graph and the polynomial. (*Source:* International Data Corporation.)

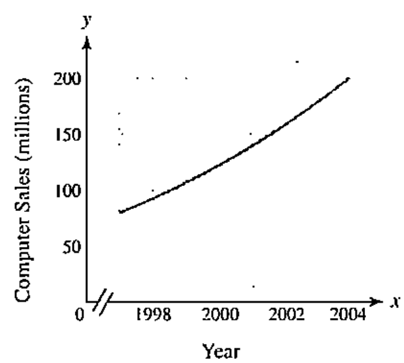


Figure 5.9 Worldwide Computer Sales

Solution

From the graph shown in Figure 5.10, it appears that personal computer sales were slightly more than 150 million, or about 160 million, in 2002. The year 2002 corresponds to $x = 5$ in the given polynomial, so substitute 5 for x and evaluate the resulting expression.

$$\begin{aligned} 0.7868x^2 + 12x + 79.5 &= 0.7868(5)^2 + 12(5) + 79.5 \\ &\approx 159 \text{ million} \end{aligned}$$

The graph and the polynomial give similiar results.

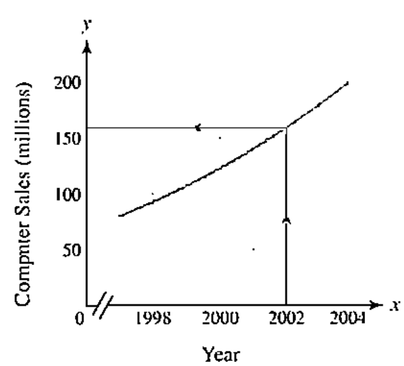


Figure 5.10 Worldwide Computer Sales

Now Try Exercise 80

5.2 PUTTING IT ALL TOGETHER

In this section we discussed monomials and polynomials, including how to add, subtract, and evaluate them. The following table summarizes several important concepts related to these topics.

Concept	Explanation	Examples
Monomial	A number, variable, or product of numbers and variables raised to natural number powers Degree is the sum of the exponents. Coefficient is the number in a monomial.	$4x^2y$ Degree: 3, coefficient: 4 $-6x^2$ Degree: 2, coefficient: -6 $-a^4$ Degree: 4, coefficient: -1 x Degree: 1, coefficient: 1 -8 Degree: 0, coefficient: -8
Polynomial	A sum of one or more monomials	$4x^2 + 8xy^2 + 3y^2$ Trinomial $-9x^4 + 100$ Binomial $-3x^2y^3$ Monomial
Like Terms	Monomials containing the same variables raised to the same powers	$10x$ and $-2x$, $4x^2$ and $3x^2$ $5ab^2$ and $-ab^2$, $5z$ and $\frac{1}{2}z$
Addition of Polynomials	To add polynomials combine like terms by applying the distributive property.	$(x^2 + 3x + 1) + (2x^2 - 2x + 7)$ $= (1 + 2)x^2 + (3 - 2)x + (1 + 7)$ $= 3x^2 + x + 8$ $3xy + 5xy = (3 + 5)xy = 8xy$
Opposite of a Polynomial	To obtain the opposite of a polynomial negate each term.	*Polynomial* / *Opposite* $-2x^2 + x - 6$ / $2x^2 - x + 6$ $a^2 - b^2$ / $-a^2 + b^2$ $-3x - 18$ / $3x + 18$
Subtraction of Polynomials	To subtract one polynomial from another, add the first polynomial to the opposite of the second polynomial.	$(x^2 + 3x) - (2x^2 - 5x)$ $= (x^2 + 3x) + (-2x^2 + 5x)$ $= (1 - 2)x^2 + (3 + 5)x$ $= -x^2 + 8x$
Evaluating a Polynomial	To evaluate a polynomial in x, substitute a value for x in the expression and simplify.	To evaluate the polynomial $3x^2 - 2x + 1$ for $x = 2$, substitute 2 for x and simplify. $3(2)^2 - 2(2) + 1 = 9$

5.2 Exercises

MyMathLab | Math XL PRACTICE | WATCH | DOWNLOAD | READ | REVIEW

CONCEPTS

1. A _____ is a number, a variable, or a product of numbers and variables raised to a natural number power.
2. The coefficient of $-3xy$ is _____.
3. A _____ is a monomial or a sum of monomials.
4. The _____ of a monomial is the sum of the exponents of the variables.
5. A polynomial with two terms is called a _____.
6. A polynomial with three terms is called a _____.
7. $4x^3 + x^2$ has _____ terms and its degree is _____.
8. To add two polynomials, combine _____ terms.
9. To subtract two polynomials, add the first polynomial to the _____ of the second polynomial.
10. Polynomials can be added horizontally or _____.

PROPERTIES OF POLYNOMIALS

Exercises 11–18: Identify the degree and coefficient of the monomial.

11. $3x^2$
12. y
13. $-ab$
14. $2xy$
15. $-5rt$
16. $8x^2y^5$
17. -6
18. $\frac{1}{2}$

Exercises 19–30: Determine whether the expression is a polynomial. If it is, state how many terms and variables the polynomial contains. Then state its degree.

19. $-x$
20. $7z$
21. $4x^2 - 5x + 9$
22. $x^3 - 9$
23. $x + \dfrac{1}{x}$
24. $\dfrac{5}{xy + 1}$
25. $3x^2y - xy^3$
26. $a^3 + 3a^2b + 3ab^2$
27. $3x^{-2}y^{-3}$
28. $5^2a^3b^4$
29. -2^3a^4bc
30. $-7y^{-1}z^{-3}$

Exercises 31–40: State whether the given pair of expressions are like terms. If they are like terms, add them.

31. $5x, -4x$
32. $x^2, 8x^2$
33. $x^3, -6x^3$
34. $4xy, -9xy$
35. $9x, -xy$
36. $5x^2y, -3xy^2$
37. ab, ba
38. $rt^2, -2t^2r$
39. $7xy^2, -3xy^2$
40. a, b

ADDITION OF POLYNOMIALS

Exercises 41–52: Add the polynomials.

41. $(3x + 5) + (-4x + 4)$
42. $(-x + 5) + (2x - 5)$
43. $(3x^2 + 4x + 1) + (x^2 + 4x - 6)$
44. $(-x^2 - x + 7) + (2x^2 + 3x - 1)$
45. $(a^3 - 6) + (4a^3 + 7)$
46. $(2b^4 - 3b^2) + (-3b^4 + b^2)$
47. $(y^3 + 3y^2 - 5) + (3y^3 + 4y - 4)$
48. $(4z^4 + z^2 - 10) + (-z^4 + 4z - 5)$
49. $(-xy + 5) + (5xy - 4)$
50. $(2a^2 + b^2) + (3a^2 - 5b^2)$
51. $(a^3b^2 + a^2b^3) + (a^2b^3 - a^3b^2)$
52. $(a^2 + ab + b^2) + (a^2 - ab + b^2)$

Exercises 53–56: Add the polynomials vertically.

53. $\begin{array}{l} 4x^2 - 2x + 1 \\ \underline{5x^2 + 3x - 7} \end{array}$
54. $\begin{array}{l} 8x^2 + 3x + 5 \\ \underline{-x^2 - 3x - 9} \end{array}$
55. $\begin{array}{l} -x^2 + x \\ \underline{2x^2 - 8x - 1} \end{array}$
56. $\begin{array}{l} a^3 - 3a^2b + 3ab^2 - b^3 \\ \underline{a^3 + 3a^2b + 3ab^2 + b^3} \end{array}$

SUBTRACTION OF POLYNOMIALS

Exercises 57–62: Write the opposite of the polynomial.

57. $5x^2$

58. $17x + 12$

59. $3a^2 - a + 4$

60. $-b^3 + 3b$

61. $-2t^2 - 3t + 4$

62. $7t^2 + t - 10$

Exercises 63–74: Subtract the polynomials.

63. $(3x + 1) - (-x + 3)$

64. $(-2x + 5) - (x + 7)$

65. $(-x^2 + 6x + 8) - (2x^2 + x - 2)$

66. $(2y^2 + 3y - 2) - (y^2 - y - 4)$

67. $(a^2 - 2a) - (4a^2 + 3a)$

68. $(7b^3 + 3b) - (-3b^3 - b)$

69. $(z^3 - 2z^2 - z) - (4z^2 + 5z + 1)$

70. $(3z^4 - z) - (-z^4 + 4z^2 - 5)$

71. $(4xy + x^2y^2) - (xy - x^2y^2)$

72. $(a^2 + b^2) - (-a^2 + b^2)$

73. $(ab^2) - (ab^2 + a^3b)$

74. $(x^2 + 3xy + 4y^2) - (x^2 - xy + 4y^2)$

Exercises 75–78: (Refer to Example 6.) Subtract the polynomials vertically.

75. $(x^2 + 2x - 3) - (2x^2 + 7x + 1)$

76. $(5x^2 - 9x - 1) - (x^2 - x + 3)$

77. $(3x^3 - 2x) - (5x^3 + 4x + 2)$

78. $(a^2 + 3ab + 2b^2) - (a^2 - 3ab + 2b^2)$

APPLICATIONS

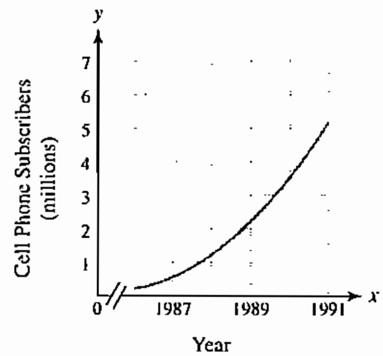
79. ***Exercise and Heart Rate*** The polynomial given by $1.6t^2 - 28t + 200$ calculates the heart rate shown in Figure 5.4(b) where t represents the elapsed time in minutes since exercise stopped.

(a) What is the heart rate when the athlete first stops exercising?

(b) What is the heart rate after 5 minutes?

(c) Describe what happens to the heart rate after exercise stops.

80. ***Cellular Phone Subscribers*** In the early years of cellular phone technology—from 1986 through 1991—the number of subscribers in millions could be modeled by the polynomial $0.163x^2 - 0.146x + 0.205$, where $x = 1$ corresponds to 1986, $x = 2$ to 1987, and so on. The graph illustrates this growth.

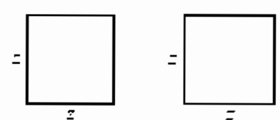


(a) Use the graph to estimate the number of cellular phone subscribers in 1990.

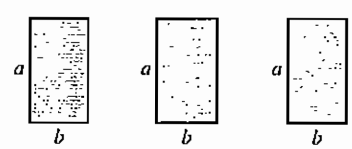
(b) Use the polynomial to estimate the number of cellular phone subscribers in 1990.

(c) Do your answers from parts (a) and (b) agree?

81. ***Areas of Squares*** Write a monomial that equals the sum of the areas of the squares. Then calculate this sum for $z = 10$ inches.



82. ***Areas of Rectangles*** Find a monomial that equals the sum of the areas of the three rectangles. Find this sum for $a = 5$ yards and $b = 3$ yards.



83. ***Area of a Figure*** Find a polynomial that equals the area of the figure. Calculate its area for $x = 6$ feet.