

① CUBIC POLYNOMIALS PROBLEM

Consider the cubic polynomial (degree 3) given by, $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

- 1a) Find the condition on the constants a, b, c so that this function has two stationary points.
- 1b) Find the x -values of the stationary points (and do not try to find the y -values).
- 1c) Find the coordinates of the point of inflexion, that is find the x and y values of the point of inflexion.
- 1d) Do a specific example with numbers substituted for a, b, c and where two stationary points occur. Draw a sketch of the cubic showing the y -intercept and the point of inflexion. You can use a calculator to get a rough idea of the y coordinates of the stationary points (should they turn out to be hard to find exactly).

[Hints: The stationary points occur when $\frac{dy}{dx} = 0$. At any point of inflexion we have $\frac{d^2y}{dx^2} = 0$.]

