1. **[5 Marks]** (Selfish and altruistic social behavior) Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.

   (a) Suppose that each person cares only about her own comfort. Model the situation as a strategic game. Is this game the Prisoner's Dilemma? Find its Nash equilibrium (equilibria?).

   (b) Suppose that each person is altruistic, ranking the outcomes according to the other person's comfort, and, out of politeness, prefers to stand than to sit if the other person stands. Model the situation as a strategic game. Is this game the Prisoner's Dilemma? Find its Nash equilibrium (equilibria?).

   (c) Compare the people's comfort in the equilibria of the two games.

2. **[5 Marks]** (Finding Nash equilibria using best response functions) Find the Nash equilibria of the two-player strategic game in which each player's set of actions is the set of nonnegative numbers and the players' payoff functions are $u_1(a_1, a_2) = a_1(a_2 - a_1)$ and $u_2(a_1, a_2) = a_2(1 - a_1 - a_2)$.

3. **[5 Marks]** (A joint project) Two people are engaged in a joint project. If each person i puts in the effort x_i, a nonnegative number equal to at most 1, which costs her $c(x_i)$, the outcome of the project is worth $f(x_1, x_2)$. The worth of the project is split equally between the two people, regardless of their effort levels. Formulate this situation as a strategic game. Find the Nash equilibria of the game when (a) $f(x_1, x_2) = 3x_1x_2$ and $c(x_i) = x_i^2$ for $i = 1, 2$, and (b) $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i = 1, 2$. In each case, is there a pair of effort levels that yields both players higher payoffs than the Nash equilibrium effort levels?

4. **[5 Marks]** (Burning a bridge) Army 1, of country 1, must decide whether to attack army 2, of country 2, which is occupying an island between the two countries. In the event of an attack, army 2 may fight, or retreat over a bridge to its mainland. Each army prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies. Model this situation as an extensive game with perfect information and show that army 2 can increase its subgame perfect equilibrium payoff (and reduce army 1's payoff) by burning the bridge to its mainland, eliminating its option to retreat if attacked.

5. (Dividing a cake fairly) Two players use the following procedure to divide a cake. Player 1 divides the cake into two pieces, and then player 2 chooses one of the pieces; player 1 obtains the remaining piece. The cake is continuously divisible (no lumps!), and each player likes all parts of it.

   (a) Suppose that the cake is perfectly homogeneous, so that each player cares only about the size of the piece of cake she obtains. How is the cake divided in a subgame perfect equilibrium?

   (b) Suppose that the cake is not homogeneous: the players evaluate different parts of it differently. Represent the cake by the set C, so that a piece of the cake is a subset P of C. Assume that if P is a subset of P' not equal to P' (smaller than P') then each player prefers P' to P. Assume also that the players' preferences are continuous: if player i prefers P to P' then there is a subset of P not equal to P that player i also prefers to P'. Let (P1,P2) (where P1 and P2 together constitute the whole cake C) be the division chosen by player 1 in a subgame perfect equilibrium of the divide-and-choose game, P2 being the piece chosen by player 2. Show that player 2 is indifferent between P1 and P2, and player 1 likes P1 at least as much as P2. Give an example in which player 1 prefers P1 to P2.