For each of the following rings answer the following questions:

1. Is it an *integral domain*?
2. Is it a *principal ideal domain (PID)*?
3. Is it a *field*?

Give reasons (i.e. short proofs, if needed) for your answer.

1. ${Z}/{13Z}$;
2. ${Z}/{20Z}$;
3. $Z×Z$ with componentwise addition and multiplication;
4. ${Q\left[X\right]}/{\left(f\right)}$ with $f=X^{2}+X+1$;
5. ${C\left[X\right]}/{\left(f\right)}$ with $f=X^{2}+X+1$;
6. $Z\left[X\right];$
7. $R\left[X\right]$ with $R={Z}/{4Z}$;
8. ${R\left[X\right]}/{I}$ with $R={Z}/{2Z}$ and$I=\left(X^{2}+X+1\right)$.