

7. Find the marginal rate of technical substitution for the isoquant for 8 units of output (a) between $(1L, 3K)$ and $(1.5L, 1.5K)$; (b) between $(1.5L, 1.5K)$ and $(3L, 1K)$; (c) at $(1.5L, 1.5K)$; and (d) at $(1L, 3K)$ and at $(3L, 1K)$. (e) What is the relevant portion of the isoquant for 8 units of output? Why?
8. (a) Starting from Figure 6-8 in the text, and assuming that both the wage of labor (w) and the rental price of capital (r) are \$2, draw a figure showing the optimal combination of labor and capital needed to produce 12 units of output. What is the capital-labor ratio at the optimal input combination? What are the total expenditures or costs of the firm required in order to produce 12 units of output with the optimal combination of labor and capital? (b) Answer the same questions as in part (a) if $w = \$1$ and $r = \$3$.
- *9. Suppose that the marginal product of the last worker employed by a firm is 40 units of output per day and the daily wage that the firm must pay is \$20, while the marginal product of the last machine rented by the firm is 120 units of output per day and the daily rental price of the machine is \$30. (a) Why is this firm not maximizing output or minimizing costs in the long run? (b) How can the firm maximize output or minimize costs?
10. John Wilson, the owner of a fast-food restaurant, estimated that he can sell 1,000 additional hamburgers per day by renting more automated equipment at a cost of \$100 per day. Alternatively, he estimated that he could sell an extra 1,200 hamburgers per day by keeping the restaurant open for two more hours per day at a cost of \$50 per hour. Which of these two alternative ways of increasing output should Mr. Wilson use?
- *11. Draw a figure similar to Figure 6-14 in the text showing constant, increasing, and decreasing returns to scale by the quantity of inputs required to double output.
12. Suppose that the production function for a commodity is given by

$$Q = 10\sqrt{LK}$$

where Q is the quantity of output, L is the quantity of labor, and K is the quantity of capital.

- (a) Indicate whether this production function exhibits constant, increasing, or decreasing returns to scale. (b) Does the production function exhibit diminishing returns? If so, when does the law of diminishing returns begin to operate? Could we ever get negative returns?
13. Indicate whether each of the following statements is true or false and give the reason. (a) A firm should stop expanding output after reaching diminishing returns and (b) if large and small firms operate in the same industry, we must have constant returns to scale.
14. (a) What is the difference between technological progress and economies of scale? (b) Suppose that technological progress is not neutral (i.e., the productivity of each input does not grow proportionately) but is labor-saving (i.e., the productivity of labor increases proportionately less than the productivity of capital). How can this type of technological progress be shown by isoquants? (c) How can we show a capital-saving innovation?

15. **Integrating Problem**

The Rapid Transit Corporation in a city has estimated the following Cobb-Douglas production function using monthly observations for the past two years:

$$\begin{aligned} \ln Q &= 2.303 + 0.40 \ln K + 0.60 \ln L \\ &\quad (3.40) \quad (4.15) \\ &\quad + 0.20 \ln G \\ &\quad (3.05) \\ R^2 &= 0.94 \quad D-W = 2.20 \end{aligned}$$

where Q is the number of bus miles driven, K is the number of buses the firm operates, L is the number of bus drivers it employs each day, and G is the gallons of gasoline it uses. The numbers in parentheses below the estimated coefficients are t values. With respect to the above results, answer the following questions: (a) Estimate Q if $K = 200$, $L = 400$, and $G = 4,000$. (b) Rewrite the estimated production function in the form of a power function. (c) Find the marginal product of capital, labor, and gasoline at $K = 200$, $L = 400$, and $G = 4,000$. (Hint: Use the formulas in footnote 10.) Are the MP_K , MP_L , and MP_G positive? Are they diminishing? Why? (d) Find the value of the output elasticity of K , L , and G . By how much does output increase by increasing