

- (1) (a) Let $h(t)$ and $V(t)$ be the height and volume of water in a cylindrical tank at time t . If water leaks through a circular hole with area a at the bottom of the tank, Torricelli's law says that the rate of change of volume is given by the equation

$$\frac{dV}{dt} = -a\sqrt{2gh},$$

where g is the acceleration due to gravity.

- (i) Show that the height of the water decreases according to the equation

$$\frac{dh}{dt} = -\alpha\sqrt{h}$$

for some constant α . What is α ?

- (ii) If we let $h(t) = u(t)^2$, find a differential equation for u .
- (iii) Given that the tank is initially full and the height of the tank is 6 metres and the radius of the tank is 2 metres, how long does it take for all the water to drain from the tank, if the hole has radius 1cm?