U. S. Census Bureau data show that 14% of the consumer population is 18 to 24 years old.

a. Given the consumer is 18 to 24 years old, what is the probability that the consumer use a plastic card?
b. Given the consumer is over 24 years old, what is the probability that the consumer uses a plastic card?
c. What is the interpretation of the probabilities shown in parts (a) and (b)?
d. Should companies such as Visa, MasterCard, and Discover make plastic cards available to the 18- to 24-year-old age group before these consumers have had time to establish a credit history? If no, why? If yes, what restrictions might the companies place on this age group?

38. A Morgan Stanley Consumer Research Survey sampled men and women and asked each whether they preferred to drink plain bottled water or a sports drink such as Gatorade or Propel Fitness water (*The Atlanta Journal-Constitution,* December 28, 2005). Suppose 200 men and 200 women participated in the study, and 280 reported they preferred plain bottled water. Of the group preferring a sports drink, 80 were men and 40 were women.

Let

M = the event the consumer is a man
W = the event the consumer is a woman
B = the event the consumer preferred plain bottled water
S = the event the consumer preferred sports drink

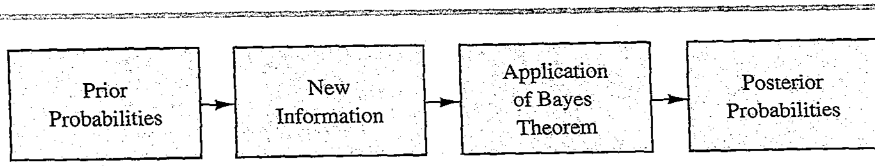
a. What is the probability a person in the study preferred plain bottled water?
b. What is the probability a person in the study preferred a sports drink?
c. What are the conditional probabilities $P(M \mid S)$ and $P(W \mid S)$?
d. What are the joint probabilities $P(M \cap S)$ and $P(W \cap S)$?
e. Given a consumer is a man, what is the probability he will prefer a sports drink?
f. Given a consumer is a woman, what is the probability she will prefer a sports drink?
g. Is preference for a sports drink independent of whether the consumer is a man or a woman? Explain using probability information.

4.5 Bayes' Theorem

In the discussion of conditional probability, we indicated that revising probabilities when new information is obtained is an important phase of probability analysis. Often, we begin the analysis with initial or **prior probability** estimates for specific events of interest. Then, from sources such as a sample, a special report, or a product test, we obtain additional information about the events. Given this new information, we update the prior probability values by calculating revised probabilities, referred to as **posterior probabilities**. **Bayes' theorem** provides a means for making these probability calculations. The steps in this probability revision process are shown in Figure 4.9.

As an application of Bayes' theorem, consider a manufacturing firm that receives shipments of parts from two different suppliers. Let A_1 denote the event that a part is from supplier 1 and A_2 denote the event that a part is from supplier 2. Currently, 65% of the parts purchased by the company are from supplier 1 and the remaining 35% are from supplier 2. Hence, if a part is selected at random, we would assign the prior probabilities $P(A_1) = .65$ and $P(A_2) = .35$.

FIGURE 4.9 PROBABILITY REVISION USING BAYES' THEOREM



Key Formulas

Counting Rule for Combinations

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{4.1}$$

Counting Rule for Permutations

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!} \tag{4.2}$$

Computing Probability Using the Complement

$$P(A) = 1 - P(A^c) \tag{4.5}$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4.6}$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \tag{4.7}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{4.8}$$

Multiplication Law

$$P(A \cap B) = P(B)P(A \mid B) \tag{4.11}$$

$$P(A \cap B) = P(A)P(B \mid A) \tag{4.12}$$

Multiplication Law for Independent Events

$$P(A \cap B) = P(A)P(B) \tag{4.13}$$

Bayes' Theorem

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \tag{4.19}$$

Supplementary Exercises

46. In a *Wall Street Journal*/Harris Interactive Personal Finance Poll, 2082 adults were asked if they owned their own home (*http://www.allbusiness.com,* January 23, 2008). Sixty percent of the respondents said yes. The percentages saying yes by age group were 26% for the 18–34 age group, 50% for the 35–44 age group, 71% for the 45–54 age group, and 88% for the 55 and over age group.
    a. What is the probability that a respondent in the 18–34 age group owns his or her own home?
    b. What is the probability that a respondent indicates he or she does not own a home?
    c. How many of the survey respondents own a home?
47. A financial manager made two new investments—one in the oil industry and one in municipal bonds. After a one-year period, each of the investments will be classified as either successful or unsuccessful. Consider the making of the two investments as an experiment.
    a. How many sample points exist for this experiment?
    b. Show a tree diagram and list the sample points.
    c. Let O = the event that the oil industry investment is successful and M = the event that the municipal bond investment is successful. List the sample points in O and in M.