Back to the TNB Future

September 1, 2010

Part I

So despite the better judgement of the upper management of Calculus III, we the writers of this lab have today decided that you and your teammates are space pirates. And being space pirates, you naturally have stolen the newly developed and very revolutionary 1.21 GW flux capacitor, designed by scientists on the planet Docbrown. Taking off in your spaceship, you realize that your crime did not go undected, and the fuzz are after you. You immediately go into evasive maneuvers, picking a flight path along the curve

$$\mathbf{r}(t) = 3 * \cos^2(t)\hat{\imath} + t^2\hat{\jmath} + 3 * \sin(t)\hat{k} \tag{1}$$

1. Plot your flight plan using *ParametricPlot3D* from "now", $t = 0$, to 3π minutes from now. Make sure to display this plot from two different viewpoints in your report.

2. The awesome computer on your ship can also generate quite a bit more information. It can not only plot your velocity, $\mathbf{v}$, as a function of time, but it can also plot the unit tangent $\hat{T}$ as well.

   - Find and plot $\mathbf{v}$ and $\hat{T}$ for $t \in [0, 3\pi]$. To find $\hat{T}$ it is useful to define $\mathbf{v}$ as vector and then use built in vector operations to determine $\hat{T}$.
   - Now that you have your picture, explain why $\mathbf{v}(t)$ keeps growing in time, but $\hat{T}(t)$ doesn't. Plot $\hat{T}$ up to 9π. What surface is $\hat{T}$ stuck on?

3. Find the arclength parameter as a function of time and plot it for $t \in [0, 3\pi]$. This integral is very hard to do! So hard, in fact, that even Mathematica gets upset at the idea. Try using the *Integrate* command in Mathematica. This will not work, an inditcation that this integral does not have a solution we can write down in terms of known functions. This is odd though, the integral we have written is perfectly well defined. Through some computational magic we can

find the numerical value of this integral. To do this in Mathematica we can use the command *NIntegrate*. Find the total distance you have traveled along your flight plan for $t \in [0, 3\pi]$.

4. Now, find $\hat{N}$, $\hat{B}$, and plot them for $t \in [0, 3\pi]$. Again, what surface are both vectors stuck on?

5. Explain why the curvature, κ, should go to zero as the time gets larger and larger. You don't have to do any math, just present a reasonable argument. Now, using Mathematica find the curvature κ. Test your hypothesis by plotting the curvature out to 6π. Do the results confirm your intuition?

6. Explain why the torsion, τ, should go to zero as the time gets larger and larger. You don't have to do any math, just present a reasonable argument. Now, using Mathematica find the torsion, τ. Test your hypothesis by plotting the torsion out to 6π. Do the results confirm your intuition?

Part II

After 2π seconds, a homing missle is launched from the surface of planet Docbrown. While being relatively sophisticated, it can't follow your path exactly. Instead, it flies along the path

$$\mathbf{r}_m(t) = 2.7\cos^2(t)\hat{\imath} + (t - 2\pi)^3\hat{\jmath} + 3\sin(t - 2\pi + .8)\hat{k} \qquad (2)$$

However, it has a proximity trigger, which is to say it only needs to get within 1 unit of you before your days as a galactic bucaneer are finished.

1. Find the time of closest approach. You will need a root finder to get the job done here. Note, one can from just looking at the equations for each flight plan why there should only be one time of closest approach. Explain why this is?

2. So instead of choosing the flight plan above, say the missle controlers had used

   $$\mathbf{r}_m(t) = 2.7\cos^2(t)\hat{\imath} + (t - 2\pi)^3\hat{\jmath} + 3\sin(t - 2\pi + .1)\hat{k}. \qquad (3)$$

   Does the missle explode in this case?