13. $_7C_7$

14. $_4C_4$

15. $_5C_0$

16. $_6C_0$

In Exercises 17–20, does the problem involve permutations or combinations? Explain your answer. (It is not necessary to solve the problem.)

17. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?

18. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is $1000, second prize is $500, and third prize is $100, in how many different ways can the prizes be awarded?

19. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

20. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is $500, in how many different ways can the prizes be awarded?

Practice PLUS

In Exercises 21–28, evaluate each expression.

21. $\dfrac{_7P_3}{3!} - _7C_3$

22. $\dfrac{_{20}P_2}{2!} - _{20}C_2$

23. $1 - \dfrac{_3P_2}{_4P_3}$

24. $1 - \dfrac{_5P_3}{_{10}P_4}$

25. $\dfrac{_7C_3}{_5C_4} - \dfrac{98!}{96!}$

26. $\dfrac{_{10}C_3}{_6C_4} - \dfrac{46!}{44!}$

27. $\dfrac{_4C_2 \cdot _6C_1}{_{18}C_3}$

28. $\dfrac{_5C_1 \cdot _7C_2}{_{12}C_3}$

Application Exercises

Use the Fundamental Counting Principle to solve Exercises 29–40.

29. The model of the car you are thinking of buying is available in nine different colors and three different styles (hatchback, sedan, or station wagon). In how many ways can you order the car?

30. A popular brand of pen is available in three colors (red, green, or blue) and four writing tips (bold, medium, fine, or micro). How many different choices of pens do you have with this brand?

31. An ice cream store sells two drinks (sodas or milk shakes), in four sizes (small, medium, large, or jumbo), and five flavors (vanilla, strawberry, chocolate, coffee, or pistachio). In how many ways can a customer order a drink?

32. A restaurant offers the following lunch menu.

Main Course	Vegetables	Beverages	Desserts
Ham	Potatoes	Coffee	Cake
Chicken	Peas	Tea	Pie
Fish	Green beans	Milk	Ice cream
Beef		Soda	

If one item is selected from each of the four groups, in how many ways can a meal be ordered? Describe two such orders.

33. You are taking a multiple-choice test that has five questions. Each of the questions has three answer choices, with one correct answer per question. If you select one of these three choices for each question and leave nothing blank, in how many ways can you answer the questions?

34. You are taking a multiple-choice test that has eight questions. Each of the questions has three answer choices, with one correct answer per question. If you select one of these three choices for each question and leave nothing blank, in how many ways can you answer the questions?

35. In the original plan for area codes in 1945, the first digit could be any number from 2 through 9, the second digit was either 0 or 1, and the third digit could be any number except 0. With this plan, how many different area codes were possible?

36. How many different four-letter radio station call letters can be formed if the first letter must be W or K?

37. Six performers are to present their comedy acts on a weekend evening at a comedy club. One of the performers insists on being the last stand-up comic of the evening. If this performer's request is granted, how many different ways are there to schedule the appearances?

38. Five singers are to perform at a night club. One of the singers insists on being the last performer of the evening. If this singer's request is granted, how many different ways are there to schedule the appearances?

39. In the *Cambridge Encyclopedia of Language* (Cambridge University Press, 1987), author David Crystal presents five sentences that make a reasonable paragraph regardless of their order. The sentences are as follows:

- Mark had told him about the foxes.
- John looked out the window.
- Could it be a fox?
- However, nobody had seen one for months.
- He thought he saw a shape in the bushes.

How many different five-sentence paragraphs can be formed if the paragraph begins with "He thought he saw a shape in the bushes" and ends with "John looked out of the window"?

40. A television programmer is arranging the order that five movies will be seen between the hours of 6 P.M. and 4 A.M. Two of the movies have a G rating and they are to be shown in the first two time blocks. One of the movies is rated NC-17 and it is to be shown in the last of the time blocks, from 2 A.M. until 4 A.M. Given these restrictions, in how many ways can the five movies be arranged during the indicated time blocks?

Use the formula for $_nP_r$ to solve Exercises 41–48.

41. A club with ten members is to choose three officers—president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?

42. A corporation has ten members on its board of directors. In how many different ways can it elect a president, vice-president, secretary, and treasurer?

43. For a segment of a radio show, a disc jockey can play 7 songs. If there are 13 songs to select from, in how many ways can the program for this segment be arranged?

44. Suppose you are asked to list, in order of preference, the three best movies you have seen this year. If you saw 20 movies during the year, in how many ways can the three best be chosen and ranked?

45. In a race in which six automobiles are entered and there are no ties, in how many ways can the first three finishers come in?

46. In a production of *West Side Story*, eight actors are considered for the male roles of Tony, Riff, and Bernardo. In how many ways can the director cast the male roles?

47. Nine bands have volunteered to perform at a benefit concert, but there is only enough time for five of the bands to play. How many lineups are possible?

48. How many arrangements can be made using four of the letters of the word COMBINE if no letter is to be used more than once?

Use the formula for $_nC_r$ to solve Exercises 49–56.

49. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

50. A four-person committee is to be elected from an organization's membership of 11 people. How many different committees are possible?

51. Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take?

52. There are 14 standbys who hope to get seats on a flight, but only 6 seats are available on the plane. How many different ways can the 6 people be selected?

53. You volunteer to help drive children at a charity event to the zoo, but you can fit only 8 of the 17 children present in your van. How many different groups of 8 children can you drive?

54. Of the 100 people in the U.S. Senate, 18 serve on the Foreign Relations Committee. How many ways are there to select Senate members for this committee (assuming party affiliation is not a factor in selection)?

55. To win at LOTTO in the state of Florida, one must correctly select 6 numbers from a collection of 53 numbers (1 through 53). The order in which the selection is made does not matter. How many different selections are possible?

56. To win in the New York State lottery, one must correctly select 6 numbers from 59 numbers. The order in which the selection is made does not matter. How many different selections are possible?

In Exercises 57–66, solve by the method of your choice.

57. In a race in which six automobiles are entered and there are no ties, in how many ways can the first four finishers come in?

58. A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection?

59. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?

60. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is $1000, second prize is $500, and third prize is $100, in how many different ways can the prizes be awarded?

61. From a club of 20 people, in how many ways can a group of three members be selected to attend a conference?

62. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is $500, in how many different ways can the prizes be awarded?

63. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

64. Nine comedy acts will perform over two evenings. Five of the acts will perform on the first evening and the order in which the acts perform is important. How many ways can the schedule for the first evening be made?

65. Using 15 flavors of ice cream, how many cones with three different flavors can you create if it is important to you which flavor goes on the top, middle, and bottom?

66. Baskin-Robbins offers 31 different flavors of ice cream. One of their items is a bowl consisting of three scoops of ice cream, each a different flavor. How many such bowls are possible?

Exercises 67–72 are based on the following jokes about books:

- *"Outside of a dog, a book is man's best friend. Inside of a dog, it's too dark to read."*—Groucho Marx
- *"I recently bought a book of free verse. For $12."* —George Carlin
- *"If a word in the dictionary was misspelled, how would we know?"*—Steven Wright
- *"Encyclopedia is a Latin term. It means 'to paraphrase a term paper.' "*—Greg Ray
- *"A bookstore is one of the only pieces of evidence we have that people are still thinking."*—Jerry Seinfeld
- *"I honestly believe there is absolutely nothing like going to bed with a good book. Or a friend who's read one."* —Phyllis Diller

67. In how many ways can these six jokes be ranked from best to worst?

68. If Phyllis Diller's joke about books is excluded, in how many ways can the remaining five jokes be ranked from best to worst?

69. In how many ways can people select their three favorite jokes from these comments about books?

70. In how many ways can people select their two favorite jokes from these comments about books?

71. If the order in which these jokes are told makes a difference in terms of how they are received, how many ways can they be delivered if George Carlin's joke is delivered first and Jerry Seinfeld's joke is told last?

72. If the order in which these jokes are told makes a difference in terms of how they are received, how many ways can they be delivered if a joke by a man is told first?

In Exercises 11–16, a die is rolled. Find the probability of getting

11. a 4. **12.** a 5.

13. an odd number.

14. a number greater than 3.

15. a number greater than 4.

16. a number greater than 7.

In Exercises 17–20, you are dealt one card from a standard 52-card deck. Find the probability of being dealt

17. a queen. **18.** a diamond.

19. a picture card.

20. a card greater than 3 and less than 7.

In Exercises 21–22, a fair coin is tossed two times in succession. The sample space of equally likely outcomes is {HH, HT, TH, TT}. Find the probability of getting

21. two heads.

22. the same outcome on each toss.

In Exercises 23–24, you select a family with three children. If M represents a male child and F a female child, the sample space of equally likely outcomes is {MMM, MMF, MFM, MFF, FMM, FMF, FFM, FFF}. Find the probability of selecting a family with

23. at least one male child.

24. at least two female children.

In Exercises 25–26, a single die is rolled twice. The 36 equally likely outcomes are shown as follows:

		Second Roll					
		⚀	⚁	⚂	⚃	⚄	⚅
First Roll	⚀	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	⚁	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	⚂	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	⚃	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	⚄	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	⚅	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Find the probability of getting

25. two numbers whose sum is 4.

26. two numbers whose sum is 6.

27. To play the California lottery, a person has to correctly select 6 out of 51 numbers, paying $1 for each six-number selection. If you pick six numbers that are the same as the ones drawn by the lottery, you win mountains of money. What is the probability that a person with one combination of six numbers will win? What is the probability of winning if 100 different lottery tickets are purchased?

28. A state lottery is designed so that a player chooses six numbers from 1 to 30 on one lottery ticket. What is the probability that a player with one lottery ticket will win? What is the probability of winning if 100 different lottery tickets are purchased?

*Exercises 29–30 involve a deck of 52 cards. If necessary, refer to the picture of a deck of cards, **Figure 12.13** on page 914.*

29. A poker hand consists of five cards.
  a. Find the total number of possible five-card poker hands.
  b. A diamond flush is a five-card hand consisting of all diamonds. Find the number of possible diamond flushes.
  c. Find the probability of being dealt a diamond flush.

30. If you are dealt 3 cards from a shuffled deck of 52 cards, find the probability that all 3 cards are picture cards.

The table shows the educational attainment of the U.S. population, ages 25 and over. Use the data in the table, expressed in millions, to solve Exercises 31–36.

Educational Attainment, in Millions, of the United States Population, Ages 25 and Over

	Less Than 4 Years High School	4 Years High School Only	Some College (Less than 4 years)	4 Years College (or More)	Total
Male	14	25	20	23	82
Female	15	31	24	22	92
Total	29	56	44	45	174

Source: U.S. Census Bureau

Find the probability, expressed as a simplified fraction, that a randomly selected American, aged 25 or over

31. has not completed four years (or more) of college.

32. has not completed four years of high school.

33. has completed four years of high school only or less than four years of college.

34. has completed less than four years of high school or four years of high school only.

35. has completed four years of high school only or is a man.

36. has completed four years of high school only or is a woman.

In Exercises 37–42, you are dealt one card from a 52-card deck. Find the probability that

37. you are not dealt a king.

38. you are not dealt a picture card.

39. you are dealt a 2 or a 3.

40. you are dealt a red 7 or a black 8.

41. you are dealt a 7 or a red card.

42. you are dealt a 5 or a black card.

In Exercises 43–44, it is equally probable that the pointer on the spinner shown will land on any one of the eight regions, numbered 1 through 8. If the pointer lands on a borderline, spin again.
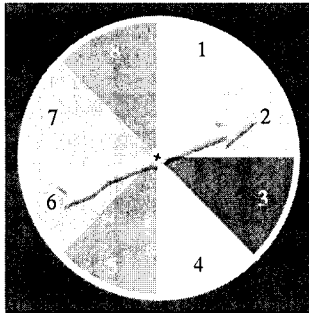
Find the probability that the pointer will stop on

43. an odd number or a number less than 6.

44. an odd number or a number greater than 3.

Use this information to solve Exercises 45–46. The mathematics department of a college has 8 male professors, 11 female professors, 14 male teaching assistants, and 7 female teaching assistants. If a person is selected at random from the group, find the probability that the selected person is

45. a professor or a male.

46. a professor or a female.

In Exercises 47–50, a single die is rolled twice. Find the probability of rolling

47. a 2 the first time and a 3 the second time.

48. a 5 the first time and a 1 the second time.

49. an even number the first time and a number greater than 2 the second time.

50. an odd number the first time and a number less than 3 the second time.

51. If you toss a fair coin six times, what is the probability of getting all heads?

52. If you toss a fair coin seven times, what is the probability of getting all tails?

53. The probability that South Florida will be hit by a major hurricane (category 4 or 5) in any single year is $\frac{1}{16}$.
(*Source*: National Hurricane Center)

a. What is the probability that South Florida will be hit by a major hurricane two years in a row?

b. What is the probability that South Florida will be hit by a major hurricane in three consecutive years?

c. What is the probability that South Florida will not be hit by a major hurricane in the next ten years?

d. What is the probability that South Florida will be hit by a major hurricane at least once in the next ten years?

54. Describe the difference between theoretical probability and empirical probability.

55. Give an example of an event whose probability must be determined empirically rather than theoretically.

56. Write a probability word problem whose answer is one of the following fractions: $\frac{1}{6}$ or $\frac{1}{4}$ or $\frac{1}{3}$.

57. Explain how to find the probability of an event not occurring. Give an example.

58. What are mutually exclusive events? Give an example of two events that are mutually exclusive.

59. Explain how to find *or* probabilities with mutually exclusive events. Give an example.

60. Give an example of two events that are not mutually exclusive.

61. Explain how to find *or* probabilities with events that are not mutually exclusive. Give an example.

62. Explain how to find *and* probabilities with independent events. Give an example.

Critical Thinking Exercises

Make Sense? *In Exercises 63–66, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.*

63. The probability that Jill will win the election is 0.7 and the probability that she will not win is 0.4.

64. Assuming the next U.S. president will be a Democrat or a Republican, the probability of a Republican president is 0.5.