## 9.4 EXERCISE SET

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### Practice Exercises

*In all exercises, assume that all variables and variable expressions represent positive numbers.*

*In Exercises 1–36, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.*

**1.** $\log_5(7 \cdot 3)$

**2.** $\log_8(13 \cdot 7)$

**3.** $\log_7(7x)$

**4.** $\log_9(9x)$

**5.** $\log(1000x)$

**6.** $\log(10{,}000x)$

**7.** $\log_7\left(\frac{7}{x}\right)$

**8.** $\log_9\left(\frac{9}{x}\right)$

**9.** $\log\left(\frac{x}{100}\right)$

**10.** $\log\left(\frac{x}{1000}\right)$

**11.** $\log_4\left(\frac{64}{y}\right)$

**12.** $\log_5\left(\frac{125}{y}\right)$

**13.** $\ln\left(\frac{e^2}{5}\right)$

**14.** $\ln\left(\frac{e^4}{8}\right)$

**15.** $\log_b x^3$

**16.** $\log_b x^7$

**17.** $\log N^{-6}$

**18.** $\log M^{-8}$

**19.** $\ln\sqrt[5]{x}$

**20.** $\ln\sqrt[7]{x}$

**21.** $\log_b(x^2y)$

**22.** $\log_b(xy^3)$

**23.** $\log_4\left(\frac{\sqrt{x}}{64}\right)$

**24.** $\log_5\left(\frac{\sqrt{x}}{25}\right)$

**25.** $\log_6\left(\frac{36}{\sqrt{x+1}}\right)$

**26.** $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$

**27.** $\log_b\left(\frac{x^2y}{z^2}\right)$

**28.** $\log_b\left(\frac{x^3y}{z^2}\right)$

**29.** $\log\sqrt{100x}$

**30.** $\ln\sqrt{ex}$

**31.** $\log\sqrt[3]{\frac{x}{y}}$

**32.** $\log\sqrt[5]{\frac{x}{y}}$

**33.** $\log_b\left(\frac{\sqrt{x}y^3}{z^3}\right)$

**34.** $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$

**35.** $\log_5\sqrt[3]{\frac{x^2y}{25}}$

**36.** $\log_2\sqrt[5]{\frac{xy^4}{16}}$

*In Exercises 37–60, use properties of logarithms to condense each logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.*

**37.** $\log 5 + \log 2$

**38.** $\log 250 + \log 4$

**39.** $\ln x + \ln 7$

**40.** $\ln x + \ln 3$

**41.** $\log_2 96 - \log_2 3$

**42.** $\log_3 405 - \log_3 5$

**43.** $\log(2x+5) - \log x$

**44.** $\log(3x+7) - \log x$

* **45.** $\log x + 3\log y$

**46.** $\log x + 7\log y$

**47.** $\frac{1}{2}\ln x + \ln y$

**48.** $\frac{1}{3}\ln x + \ln y$

**49.** $2\log_b x + 3\log_b y$

**50.** $5\log_b x + 6\log_b y$

* **51.** $5\ln x - 2\ln y$

**52.** $7\ln x - 3\ln y$

**53.** $3\ln x - \frac{1}{3}\ln y$

**54.** $2\ln x - \frac{1}{2}\ln y$

**55.** $4\ln(x+6) - 3\ln x$

**56.** $8\ln(x+9) - 4\ln x$

**57.** $3\ln x + 5\ln y - 6\ln z$

**58.** $4\ln x + 7\ln y - 3\ln z$

* **59.** $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

**60.** $\frac{1}{3}(\log_4 x - \log_4 y) + 2\log_4(x+1)$

*In Exercises 61–68, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.*

**61.** $\log_5 13$

**62.** $\log_6 17$

**63.** $\log_{14} 87.5$

**64.** $\log_{16} 57.2$

**65.** $\log_{0.1} 17$

**66.** $\log_{0.3} 19$

**67.** $\log_\pi 63$

**68.** $\log_\pi 400$

*Solve each logarithmic equation in Exercises 41–90. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer. Then, where necessary, use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.*

**41.** $\log_3 x = 4$

**42.** $\log_5 x = 3$

**43.** $\log_2 x = -4$

**44.** $\log_2 x = -5$

**45.** $\log_9 x = \frac{1}{2}$

**46.** $\log_{25} x = \frac{1}{2}$

**47.** $\log x = 2$

**48.** $\log x = 3$

**49.** $\log_4(x + 5) = 3$

**50.** $\log_5(x - 7) = 2$

**51.** $\log_3(x - 4) = -3$

**52.** $\log_7(x + 2) = -2$

**53.** $\log_4(3x + 2) = 3$

**54.** $\log_2(4x + 1) = 5$

**55.** $\ln x = 2$

**56.** $\ln x = 3$

**57.** $\ln x = -3$

**58.** $\ln x = -4$

**59.** $5\ln(2x) = 20$

**60.** $6\ln(2x) = 30$

**61.** $6 + 2\ln x = 5$

**62.** $7 + 3\ln x = 6$

**63.** $\ln\sqrt{x + 3} = 1$

**64.** $\ln\sqrt{x + 4} = 1$

**65.** $\log_5 x + \log_5(4x - 1) = 1$

**66.** $\log_6(x + 5) + \log_6 x = 2$

**67.** $\log_3(x - 5) + \log_3(x + 3) = 2$

**68.** $\log_2(x - 1) + \log_2(x + 1) = 3$

**69.** $\log_2(x + 2) - \log_2(x - 5) = 3$

**70.** $\log_4(x + 2) - \log_4(x - 1) = 1$

**71.** $\log(3x - 5) - \log(5x) = 2$

**72.** $\log(2x - 1) - \log x = 2$

**73.** $\ln(x + 1) - \ln x = 1$

**74.** $\ln(x + 2) - \ln x = 2$

**75.** $\log_3(x + 4) = \log_3 7$

**76.** $\log_2(x - 5) = \log_2 4$

**77.** $\log(x + 4) = \log x + \log 4$

**78.** $\log(5x + 1) = \log(2x + 3) + \log 2$

**79.** $\log(3x - 3) = \log(x + 1) + \log 4$

**80.** $\log(2x - 1) = \log(x + 3) + \log 3$

**81.** $2\log x = \log 25$

**82.** $3\log x = \log 125$

**83.** $\log(x + 4) - \log 2 = \log(5x + 1)$

**84.** $\log(x + 7) - \log 3 = \log(7x + 1)$

**85.** $2\log x - \log 7 = \log 112$

**86.** $\log(x - 2) + \log 5 = \log 100$

**87.** $\log x + \log(x + 3) = \log 10$

**88.** $\log(x + 3) + \log(x - 2) = \log 14$

**89.** $\ln(x - 4) + \ln(x + 1) = \ln(x - 8)$

**90.** $\log_2(x - 1) - \log_2(x + 3) = \log_2\left(\frac{1}{x}\right)$

## Practice PLUS

*In Exercises 91–98, solve each equation.*

**91.** $5^{2x} \cdot 5^{4x} = 125$

**92.** $3^{x+2} \cdot 3^x = 81$

**93.** $3^{x^2} = 45$

**94.** $5^{x^2} = 50$

**95.** $\log_2(x - 6) + \log_2(x - 4) - \log_2 x = 2$

**96.** $\log_2(x - 3) + \log_2 x - \log_2(x + 2) = 2$

**97.** $5^{x^2-12} = 25^{2x}$

**98.** $3^{x^2-12} = 9^{2x}$

## Application Exercises

**99.** The formula $A = 36.1e^{0.0126t}$ models the population of California, $A$, in millions, $t$ years after 2005.

a. What was the population of California in 2005?

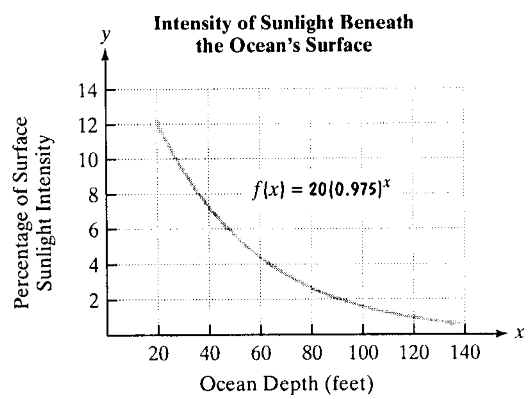
b. When will the population of California reach 40 million?

**100.** The formula $A = 22.9e^{0.0183t}$ models the population of Texas, $A$, in millions, $t$ years after 2005.

a. What was the population of Texas in 2005?

b. When will the population of Texas reach 27 million?

*The function $f(x) = 20(0.975)^x$ models the percentage of surface sunlight, $f(x)$, that reaches a depth of x feet beneath the surface of the ocean. The figure shows the graph of this function. Use this information to solve Exercises 101–102.*




**101.** Use the function to determine at what depth, to the nearest foot, there is 1% of surface sunlight. How is this shown on the graph of $f$?

**102.** Use the function to determine at what depth, to the nearest foot, there is 3% of surface sunlight. How is this shown on the graph of $f$?

## 9.6 EXERCISE SET MyMathLab

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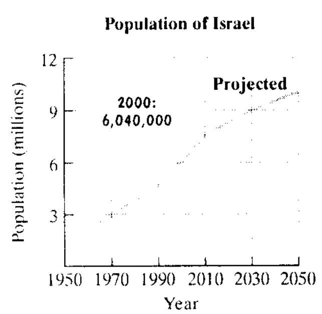
### Practice Exercises and Application Exercises

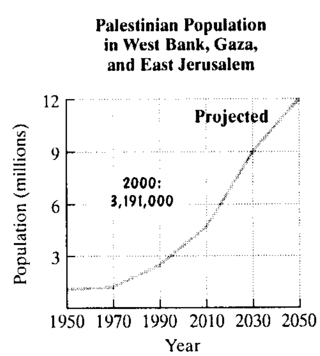
*The exponential models describe the population of the indicated country, A, in millions, t years after 2006. Use these models to solve Exercises 1–6.*

| | |
|---|---|
| India | $A = 1095.4e^{0.014t}$ |
| Iraq | $A = 26.8e^{0.027t}$ |
| Japan | $A = 127.5e^{0.001t}$ |
| Russia | $A = 142.9e^{-0.004t}$ |

1. What was the population of Japan in 2006?
2. What was the population of Iraq in 2006?
3. Which country has the greatest growth rate? By what percentage is the population of that country increasing each year?
4. Which country has a decreasing population? By what percentage is the population of that country decreasing each year?
5. When will India's population be 1238 million?
6. When will India's population be 1416 million?

*About the size of New Jersey, Israel has seen its population soar to more than 6 million since it was established. With the help of U.S. aid, the country now has a diversified economy rivaling those of other developed Western nations. By contrast, the Palestinians, living under Israeli occupation and a corrupt regime, endure bleak conditions. The graphs show that by 2050, Palestinians in the West Bank, Gaza Strip, and East Jerusalem will outnumber Israelis. Exercises 7–8 involve the projected growth of these two populations.*




*Source:* Newsweek

7. a. In 2000, the population of Israel was approximately 6.04 million and by 2050 it is projected to grow to 10 million. Use the exponential growth model $A = A_0e^{kt}$, in which $t$ is the number of years after 2000, to find an exponential growth function that models the data.

   b. In which year will Israel's population be 9 million?

8. a. In 2000, the population of the Palestinians in the West Bank, Gaza Strip, and East Jerusalem was approximately 3.2 million and by 2050 it is projected to grow to 12 million. Use the exponential growth model $A = A_0e^{kt}$, in which $t$ is the number of years after 2000, to find an exponential growth function that models the data.

   b. In which year will the Palestinian population be 9 million?

*An artifact originally had 16 grams of carbon-14 present. The decay model $A = 16e^{-0.000121t}$ describes the amount of carbon-14 present after t years. Use this model to solve Exercises 9–10.*

9. How many grams of carbon-14 will be present in 5715 years?
10. How many grams of carbon-14 will be present in 11,430 years?

*

11. The half-life of the radioactive element krypton-91 is 10 seconds. If 16 grams of krypton-91 are initially present, how many grams are present after 10 seconds? 20 seconds? 30 seconds? 40 seconds? 50 seconds?
12. The half-life of the radioactive element plutonium-239 is 25,000 years. If 16 grams of plutonium-239 are initially present, how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

*Use the exponential decay model for carbon-14, $A = A_0e^{-0.000121t}$, to solve Exercises 13–14.*

13. Prehistoric cave paintings were discovered in a cave in France. The paint contained 15% of the original carbon-14. Estimate the age of the paintings.
14. Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in a living person. In 1989, how old were the skeletons?
15. The August 1978 issue of *National Geographic* described the 1964 find of bones of a newly discovered dinosaur weighing 170 pounds, measuring 9 feet, with a 6-inch claw on one toe of each hind foot. The age of the dinosaur was estimated using potassium-40 dating of rocks surrounding the bones.

   a. Potassium-40 decays exponentially with a half-life of approximately 1.31 billion years. Use the fact that after 1.31 billion years a given amount of potassium-40 will have decayed to half the original amount to show that the decay model for potassium-40 is given by $A = A_0e^{-0.52912t}$, where $t$ is in billions of years.

   b. Analysis of the rocks surrounding the dinosaur bones indicated that 94.5% of the original amount of potassium-40 was still present. Let $A = 0.945A_0$ in the model in part (a) and estimate the age of the bones of the dinosaur.

16. A bird species in danger of extinction has a population that is decreasing exponentially ($A = A_0e^{kt}$). Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

17. Use the exponential growth model, $A = A_0e^{kt}$, to show that the time it takes a population to double (to grow from $A_0$ to $2A_0$) is given by $t = \frac{\ln 2}{k}$.

18. Use the exponential growth model, $A = A_0e^{kt}$, to show that the time it takes a population to triple (to grow from $A_0$ to $3A_0$) is given by $t = \frac{\ln 3}{k}$.

*Use the formula $t = \frac{\ln 2}{k}$ that gives the time for a population with a growth rate k to double to solve Exercises 19–20. Express each answer to the nearest whole year.*

19. The growth model $A = 4.1e^{0.01t}$ describes New Zealand's population, $A$, in millions, $t$ years after 2006.
   a. What is New Zealand's growth rate?
   b. How long will it take New Zealand to double its population?

20. The growth model $A = 107.4e^{0.012t}$ describes Mexico's population, $A$, in millions, $t$ years after 2003.
   a. What is Mexico's growth rate?
   b. How long will it take Mexico to double its population?

*Exercises 21–26 present data in the form of tables. For each data set shown by the table,*

a. *Create a scatter plot for the data.*
b. *Use the scatter plot to determine whether an exponential function, a logarithmic function, or a linear function is the best choice for modeling the data. (If applicable, in Exercise 45 you will use your graphing utility to obtain these functions.)*

**21. Percent of Miscarriages, by Age**

| [illegible] | Percent of Miscarriages |
|---|---|
| 22 | 9% |
| 27 | 10% |
| 32 | 13% |
| 37 | 20% |
| 42 | 38% |
| 47 | 52% |

*Source:* Time

**22. Savings Needed for Health-Care Expenses during Retirement**

| Age at Death | Savings Needed |
|---|---|
| 80 | \$219,000 |
| 85 | \$307,000 |
| 90 | \$409,000 |
| 95 | \$524,000 |
| 100 | \$656,000 |

*Source:* Employee Benefit Research Institute

**23. Intensity and Loudness Level of Various Sounds**

| Intensity (watts per meter$^2$) | Loudness Level (decibels) |
|---|---|
| 0.1 (loud thunder) | 110 |
| 1 (rock concert, 2 yd from speakers) | 120 |
| 10 (jackhammer) | 130 |
| 100 (jet takeoff, 40 yd away) | 140 |

**24. Temperature Increase in an Enclosed Vehicle**

| Minutes | Temperature Increase (°F) |
|---|---|
| 10 | 19° |
| 20 | 29° |
| 30 | 34° |
| 40 | 38° |
| 50 | 41° |
| 60 | 43° |

**25. U.S. Per Capita Consumption of Bottled Water**

| Year | Per Capita Consumption (gallons) |
|---|---|
| 2001 | 18.8 |
| 2002 | 20.9 |
| 2003 | 22.4 |
| 2004 | 24.0 |
| 2005 | 26.1 |
| 2006 | 28.3 |

*Source:* Beverage Marketing Corporation