



1. The Rosenbrock function

$$f(x, y) = (1 - x)^2 + 100(y - x^2)^2$$

(shown above) is a well-known test function for optimization methods.

Local maxima and minima for this function will occur where $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) = (0, 0)$, i.e. if

$$\begin{aligned} E_1(x, y) &= 0, \quad \text{and} \\ E_2(x, y) &= 0 \end{aligned}$$

where $E_1 = \frac{\partial f}{\partial x}$ and $E_2 = \frac{\partial f}{\partial y}$.

This system of nonlinear algebraic equations may be iteratively solved using Newton's method. If we start with an initial guess (x_0, y_0) each new iterate is found using the following matrix update equation:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - \begin{bmatrix} \frac{\partial E_1}{\partial x}(x_n, y_n) & \frac{\partial E_1}{\partial y}(x_n, y_n) \\ \frac{\partial E_2}{\partial x}(x_n, y_n) & \frac{\partial E_2}{\partial y}(x_n, y_n) \end{bmatrix}^{-1} \begin{bmatrix} E_1(x_n, y_n) \\ E_2(x_n, y_n) \end{bmatrix}$$

Your task is to write a C program which implements this 2-D Newton's method to find the minimum of the Rosenbrock function, starting from the point $(-2, 2)$. The functions $E_1(x, y)$ and $E_2(x, y)$ and their x and y partial derivatives should all be coded as functions outside `main()`.

(Hint: You should use the fact that the inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.)