

(1) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F}(x, y) = (1 + 2xy)\mathbf{i} + (1 + x^2)\mathbf{j}$ and C is the curve parameterised by

$$\mathbf{r}(t) = t\mathbf{i} + \sin\left(\frac{\pi}{2}t^2\right)\mathbf{j}, \quad 0 \leq t \leq 1.$$

(2) Let C be a circle of radius 1 centred at the point $(x, y) = (1, 0)$. Consider the vector field $\mathbf{F}(x, y) = x\mathbf{i} + x(1 + y)\mathbf{j}$. Calculate the two line integrals over C ,

$$\oint_C \mathbf{F} \cdot \mathbf{T} \, ds \quad \text{and} \quad \oint_C \mathbf{F} \cdot \mathbf{n} \, ds,$$

where \mathbf{T} is the unit tangent vector to C traversed in an anticlockwise direction, and \mathbf{n} is a unit normal vector to C directed away from the centre of the circle.

(3) Evaluate the line integral

$$\oint_C ((y^2 - x)dx + (3x + y)dy)$$

where C is the boundary of the region in the x - y plane enclosed by the parabola $y = x^2$ and the line $y = 1$, traversed in an anticlockwise direction.

(4) Find the area of the part of paraboloid $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 1$.
