with the MR_A curve. The best level of output of both beef and hides is 40 units and is given by point E, at which the MC curve for cattle (both beef and hides together) crosses the MR_T curve of the firm. At $Q = 40$, $P_A = \$12$ on the D_A curve and $P_B = \$5$ on the D_B curve.

In the left panel of Figure 11-2, both MR_A and MR_B are positive at the best level of output of $Q = 40$. In contrast, in the right panel of Figure 11-2, MR_B is negative at the best level of output of $Q = 60$ given by point E', at which the lower MC' curve crosses the same MR_T curve. This means that selling more than 45 units of product B (hides) reduces the firm's total revenue and profits. In such a case the firm produces 60 units of the joint product (cattle), sells 60 units of product A (beef) at $P'_A = \$10$ but sells only 45 units of product B (hides) at $P'_B = \$4.50$ (at which TR_B is maximum and $MR_B = 0$). That is, the firm withholds from the market and disposes of the extra 15 units of product B jointly produced with the 60 units of product A in order not to sell them at a negative marginal revenue.[3] An example of this was provided by the destruction of excess pineapple juice that jointly resulted from the production of sliced pineapples for canning. Until use was found for it, the excess pineapple juice was simply destroyed in order not to depress its price below the point at which its marginal revenue became negative.[4]

Optimal Pricing and Output of Joint Products Produced in Variable Proportions

Although the case of products that are produced jointly in fixed proportions (i.e., that are complementary in production) is possible, more common is the case of products that are jointly produced in variable proportions (i.e., that are substitutes in production). We can determine the profit-maximizing combination of products that are jointly produced in variable proportions with the aid of Figure 11-3.

In Figure 11-3, the curved lines are product transformation curves and show the various combinations of products A and B that the firm can produce at each level of input use and total cost. For example, the lowest curve shows that with $TC = \$100$, the firm can produce 40 units of product A and 60 units of product B (point G), 20 units of product A and 80 units of product B (point H), or any combination of products A and B shown on the curve. Higher product transformation curves refer to the various larger combinations of products A and B that can be produced at each higher level of TC. Production transformation curves are concave to the origin because the firm's production resources are not perfectly adaptable in (i.e., cannot be perfectly transferred between) the production of products A and B.

Figure 11-3 also shows *isorevenue lines.* They represent all combinations of outputs of products A and B that generate the same total revenue for the firm. For example, the lowest isorevenue line shows all the combinations of products A and B that lead to $TR = \$120$ with $P_A = \$1.50$ and $P_B = \$1.00$. For example, at point G (40A, 60B),

[3] For the mathematical analysis of optimal pricing of joint products produced in fixed proportions, see Problem 3, with answer at the end of the book.

[4] When the firm incurs a significant cost in disposing of the excess quantity of a jointly produced product, the cost is added to the MC function of the firm, thereby reducing the optimal level of output. Significant disposal costs, however, provide strong incentives for the firm to find uses for the excess products that are jointly produced.