

Factoring a quadratic with leading coefficient greater than 1

Factor.

$$5x^2 + 22x + 8$$

We'll factor $5x^2 + 22x + 8$ by using the FOIL method backward.

The coefficient of x^2 is 5. Using whole numbers, only **1** and **5** can be multiplied to get 5.

So we must find two integers p and q such that

$$5x^2 + 22x + 8 = (1x + p)(5x + q).$$

Using the FOIL method to expand the right-hand side of this equation, we get that

$$pq = 8 \text{ and } q + 5p = 22.$$

Below are all the possible integers p and q such that $pq = 8$. We check to see when $q + 5p = 22$.

p	q	$q + 5p$
1	8	13
-1	-8	-13
8	1	41
-8	-1	-41
2	4	14
-2	-4	-14
4	2	22
-4	-2	-22

From the table, we find that $p=4$ and $q=2$ work.

$$\begin{aligned} 5x^2 + 22x + 8 &= (x + p)(5x + q) \\ &= (x + 4)(5x + 2) \end{aligned}$$

-1	-40	-41
2	20	22
-2	-20	-22
4	10	14
-4	-10	-14
5	8	13
-5	-8	-13

We find that the numbers 2 and 20 work: their product is 40 and their sum is 22 .

We can use these two numbers to factor $5x^2 + 22x + 8$. Since $2 + 20 = 22$, we rewrite

$$5x^2 + 22x + 8$$

as

$$5x^2 + (2 + 20)x + 8 .$$

We then factor by grouping.

$$\begin{aligned}
 5x^2 + (2 + 20)x + 8 &= 5x^2 + 2x + 20x + 8 \\
 &= x(5x + 2) + 4(5x + 2) && \text{(Gathering terms and factoring out common factors)} \\
 &= (x + 4)(5x + 2) && \text{(Factoring out } 5x + 2)
 \end{aligned}$$

The answer is $(x + 4)(5x + 2)$.

We check our answer by applying FOIL.

$$\begin{aligned}
 (x + 4)(5x + 2) &= 5x^2 + 2x + 20x + 8 \\
 &= 5x^2 + 22x + 8 .
 \end{aligned}$$