Given the Probability Axioms:

Nonnegativity: P(A)  0 for every event A

Additivity: if all Ai are disjoint, then: P(A1  A2 …) = P(A1) + P(A2) + …

Normalization: P() = 1

**1**. Let A and B be independent events with AC denoting the complement of A. Prove that AC is

independent of B. (Remember that P(AC) = 1-P(A))

2. Regardless of whether A1;A2;A3 are mutually exclusive or not, P[A1  A2  A3]  P[A1]+

P[A2] + P[A3]. Prove this using the Axioms of Probabilities.

3. At UCON, the three most popular social networks are Facebook, Friendster, and MySpace. Let's say that we know the following about student usage of these networks:

Network % of Students with Account

Facebook 75%

Friendster 30%

Myspace 65%

40% of students have accounts in exactly 2 of the networks.

20% have accounts in all of the networks.

Given this information what percentage of students do not have an account in any of the net-

works?

4. Roughly 40 percent of all e-mail traffic in the United States is spam. With probability 0:25,

these e-mails contain the keyword "sale" in their subject. However, non-spam e-mails may also

contain that keyword but with probability 0:02. Given that you receive an email containing

"sale" in its subject, what is the probability that it is NOT a spam e-mail?

5. The disk with your favorite MP3 song has been scratched very badly. The disc was mixed up with three other scratched discs that were lying around. It is equally likely that any of the four discs holds your favorinte song. A DJ friend of yours offers to take a look since he claims to recover songs from any disc with a 50% chance (assuming the song is there). Given that he searches on disc 1 but cannot recover your song, what is the probability that your song is on disc i for i = 1; 2; 3; 4 ?

6. The company Storage4U works in data warehousing and has 3 data centers. They used to be

the bedrock for all of your file storing needs. However, with the recent economic downturn,

Storage4U has been forced to cut back and have decided that uptime is no longer important. As

a result, each of their data centers now has a probability of 0.8 to be up and running. Suppose

that in order for each file to be stored, it is broken into 3 segments and stored into each of the

3 data centers.

\_ Let Eij be the event that the jth data center is up when the file with index i is retrieved.

\_ Assume that each Eij are mutually independent events.

Out of five files attempted to be retrieved at various times, what is the probability that at least

3 are successfully retrieved?

7. An alphabet of 50 symbols can be used for transmitting messages in a communication system.

(a) How many distinct messages (sequence of symbols) of 25 symbols can the transmitter

generate if symbols can be repeated in the message?

(b) In (a), how many messages of 25 symbols can be generated if 20 of the 50 symbols can

appear only as the first and/or last symbols of the message, the other 30 symbols can

appear anywhere, and repetitions of all symbols are allowed?

(c) Repeat (a) and (b) for the case that no repetition of any symbol is allowed (i.e., if a symbol

is used in the message, it is used only once).