It's pretty clear. $|\wp(\mathbb{N}| = 2^\omega$ through a simple bijection. We know through Cantor Diagonalization that this is strictly larger than ω, thus by definition of ω_1(as the first cardinal after ω), $\omega_1 \leq 2^\omega$. Then I'm not sure why more of an argument would be necessary.

EDIT: To clarify a surjection would just be the identity until $\omega_1$as $\omega_1 \subseteq \kappa = 2^\omega$. The rest of the elements can all map to 0.

Alternatively:

Biject $\mathbb{N}$ with $\mathbb{N} \times \mathbb{N}$, Mostowski collapse well-orderings, and for those that are not WO do whatever you please (as long as you hit the finite ordinals).

Alternatively:

N == N x N (without choice) so it suffices to find a surjection
f : P(N x N) --> omega_1.

If A subset N x N is a well-order on N, f(A) is its order type.
Otherwise f(A) = 0.
(being a well-order is a "formula", so a well-defined notion)

This is a surjection, as omega_1 = the set of all countable ordinals
== all order types of well-orders on N