The sequence of discrete random variables X_n, with mass functions f_n, is said to converge in total variation to X with mass function f if

$$\sum_x |f_n(x) - f(x)| \to 0 \text{ as } n \to \infty.$$

Suppose that $X_n \to X$ in total variation and $u : \mathbb{R} \to \mathbb{R}$ is bounded. Show that

$$\lim_{n \to \infty} E[u(X_n)] \to E[u(X)]$$