

Change of Variables and Statistics

If X_1, X_2, \dots, X_n are (*iid*), from a distribution with mean μ and variance σ^2 . Define the sample mean as

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}.$$

(a) Show that the mean and variances of the probability density function of \bar{X} are given as

$$E(\bar{X}) = \mu$$

$$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

b) What is the central limit theorem?

c) If n , is large, can you describe fully, the probability density function of \bar{X} ?

d) Can you describe fully the probability density function of the variable $y = e^{\bar{X}}$? This random variable is called a lognormal random variable, and is used very frequently in finance.