***Example 1:*** Customers arrive to a store with one server at an average rate of 15 per hour (Poisson). Service time is approximately 3 minutes (assume exponential service time).

1. What is the probability that the sales person is idle?
2. What is the average number of customers waiting for service?
3. What is the average customer’s time in system?
4. What is the daily mean revenue of this store if an average customer spends $10 (assume 10 hours a day)?

**Solution:**

This is M/M/1 with  = 15/hour and  = 3 minutes= 3/60 hours. Therefore, we use the “MMm” spreadsheet and set cells E2-D4 are 15, 20, and 1.

* 1. 1-utilization = 1 - .75 = .25.
	2. Lq =2.25.
	3. Ws = 0.20 hours.
	4. The teller can serve 20 people per hour, but on average, only 15 arrive per hour. The teller thus services 15 people per hour on average. Hence,

Daily average revenue=15\*10\*10=$1500.

***Example 2:*** Sea-View Bank is a new startup bank that has only one teller. During peak demand periods, bank officials have noticed that an average of six customers come to the bank per hour and that the arrivals appear to follow a Poisson distribution. It takes the teller an average of four minutes to serve each customer (following an exponential distribution).

In response to their competition, Sea-View bank officials have decided to institute a new policy that will pay customers a bonus of $5 if they have to wait more than 4 minutes in line. Approximately how much will Sea-View Bank have to pay per hour in bonuses to their customers?

***Solution:***

This is M/M/1 with  = 6/hour and =15/hoursince average service time  = 4 minutes = 4/60 hours). In the “MMm” spreadsheet we set cells E2-D4 are 6, 15, and 1.

To find the probability that a customer will have to wait more than 4 minutes in queue, we set cell E22 to be “=4/60”. (This may have tripped up some people: we used “hours” for  and , so we must use “hours” here as well. 4 minutes = 4/60 hours.)

So, from cell E25, =0.2195.

Average total payment to customers/hour

= 5 \*  \* Prob[waiting in queue for more than 4 minutes]

= 5 \* \* 0.2195 = **$6.59/hour**.  ***Example 3:*** The law firm of LD and Associates specializes in the practice of waste disposal law. Data regarding the cases received in a year and the times to complete a case was recently compiled. Suppose that the firm works on one case at a time. Cases are received according to a Poisson distribution with a mean of one case every 30 days. The data on the time on the completion times of the last 10 cases are 27, 26, 26, 25, 27, 24, 27, 23, 22 and 23. Assuming that cases are handled one at a time on a first come first served basis, determine the number of clients waiting for their case to be processed and the average time a client has to wait until his/her case is completed (wait in queue plus service).

***Solution:***

Here we have an M/G/1 queue with:

 1/ = 30 days =>  =1/30 per day,

The average and standard deviation of service time are not given directly. We need to compute them from the given data:

 1/ = (27+26+26+25+27+24+27+23+22+23)/10 = 25 days => = 1/25 per day

 =3.55, where xi is duration of case i. Note that this is the variance. To find the standard deviation, you take the square root.

Any statistical software should be able to calculate the average, the standard deviation, and the variance for you. You can also use Microsoft Excel. The built-in functions are “=average(range),” “=stdev(range),” and “=var(range)” respectively, where “range” is where you input the raw data.

We will use the “MG1” spreadsheet and set cell E2 to be “=1/30”, cell E4 to be 25, and cell E5 to be “=sqrt(3.55)” (this calculates the square root), respectively. The question is asking for we get Lq and Ws. Note that you do **NOT** need to set cell E6 (**) – it will be automatically calculated for you from cell E4.

* From cell F10, Lq = Ave. number of cases waiting to be processed = **2.10** cases.
* From cell F13, Ws = Ave. time to complete a case = **88** days.

***Example 4:*** There is currently one tollbooth at one of the exits of a state turnpike. On the average, it takes approximately 40 seconds for the toll collector to take the money from a driver and, if necessary, return change; data analysis indicates that these service times follow an exponential distribution. Cars arrive at the tollbooth at an average rate of 70 cars per hour (assume a Poisson distribution).

1. What is the capacity utilization of the toll collector?
2. What is the average waiting time for a car before it pays the toll?
3. The turnpike authority has decided to install another toll collection station that is equipped with an electronic scanner that will automatically scan stickers on the windshields of cars that have signed up with this service. The scanning time is estimated to be 5 seconds (which is constant for all cars with a sticker). Once the new tollbooth is installed, it is estimated that 40 percent of all cars coming through the tollbooth will buy and use the stickers. What will be the average waiting time for cars with stickers and for cars without stickers?

**Solution:**

This is an M/M/1 system (a single tollbooth) with** = 70 /hour, and **= 90 /hour. We will use the “MMm” spreadsheet and set cells E2-E4 to be 70, 90, and 1.

1. Utilization = 77.78%
2. 
3. Cars with stickers:



 Because the service time is constant, it is an M/G/1 system with =0. So we will use the “MG1” spreadsheet and set cells E2, E4, and E5 to be “=28/60”, “=5/60”, and 0. From cell F12, we get  (almost no wait in queue! – but remember, it still spends 5 seconds in process).

Cars without stickers:

** = 70 \* 60% = 42 /hour, and **is still 90 /hour. As in part a)-b), we will use the “MMm” spreadsheet and set cells E2-E4 to be 42, 90, and 1. From cell F10, we get ***Example 5:*** The Airport in Upper Slobbovia (which has only one runway) is beginning to experience congestion. At the present time, an average of 12 planes arrive every hour (the actual number of arriving planes follow a Poisson distribution). With the present equipment, the controllers can land and clear a plane in an average of 4 minutes (actual service times follow exponential distribution). The planes are processes on a first come first served basis; planes that are waiting to land are asked to circle the airport.

1. What is the expected number of airplanes circling the airport at any point in time waiting for clearance to land?
2. A new ground approach radar just approved by FAA is being considered; if adopted, it will allow planes to be processed exactly every four minutes. If this radar is used, what is the expected number of airplanes that would be circling the airport at any point in time?
3. The new ground approach radar will cost $100 per hour to operate and maintain. If the average cost of keeping a plane airborne is $70 per hour, would you recommend that the radar be adopted?

***Solution:***

1. With the current setup, the landing time follows an exponential distribution, so it is an M/M/1 system. λ = 12 planes per hour, μ =15 planes per hour. We use “MMm” spreadsheet and set cells E2-E4 to be 12, 15, and 1. Then,

Lq = 3.2 planes.

1. With the new radar, the landing time is a constant 4 minutes. So this is a M/G/1 system. λ = 12 planes per hour, 1/μ =4 minutes = (4/60) hours. Standard deviation of service time = 0. We use the “MG1” spreadsheet and set cells E2, E4, and E5 to be 12, “=4/60”, and 0. Then,

Lq = 1.6 planes

(Notice that by simply reducing the variability we are able to cut the queue by half).

To calculate waiting cost per hour, there are two approaches:

* For each airplane, the average waiting time in queue (waiting while airborne) is Wq, so the average cost of waiting is $70\*Wq. However, for the whole system, there are on average λ planes every hour, so the average hourly waiting cost is $70\* Wq\* λ.
* From the system perspective, to calculate the **average** waiting cost, it’s almost as if there are Lq planes waiting to land at any time. Since each plane costs $70 per hour to keep airborne, the total system hourly waiting cost is $70\*Lq.

The two actually are the same, because the Little’s Law says Lq= λ\*Wq.

1. With the old system, waiting cost per hour=70 Lq=$224/hour.
2. With the new system, waiting cost per hour=70 Lq=$112/hour.

New radar operating cost per hour = $100/hour.

Therefore, we would recommend that the radar be adopted ($212<$224). Note that the cost difference per hour is small, but over a long stretch of time, it is significant.

***Example 6:*** To promote its reputation for fast service, Earl’s While-U-Wait Automotive Tune-up Shop promises to reduce a customer’s bill by $0.20 for every minute the customer must wait until his or her car’s tune-up is finished. Customers arrive according to a Poisson distribution with a mean arrival rate of 5 customers per hour. The time required by a mechanic to perform a tune-up is exponentially distributed with an average time of 30 minutes.

Earl pays his mechanics $20 per hour. If Earl wants to minimize his total costs (defined as the sum of his labor costs and the profit lost per hour because of reductions in customer’s bills), how many mechanics should Earl hire?

***Solution:***

This is an M/M/m system where the number of servers, *m*, needs to be determined. λ = 5 customers per hour, μ = 2 customers per hour. Server cost is $20 per hour, and waiting cost is $12 per hour. We use the “MMm” spreadsheet and set cells E2-E3 to be 5 and 2. Then we vary cell E4 (first column in the following table):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| # of mechanics (m) | Cost of mechanics ($20\*m) | Avg. # of customers in the system (Ls)  | Cost of customer waiting ($12\*Ls) | Expected total cost per hour |
| 3 | 60 | 6.0112 | 72.1344 | 132.1344 |
| **4** | **80** | **3.0331** | **36.3972** | **116.3972** |
| 5 | 100 | 2.6304 | 31.5648 | 131.5648 |
| 6 | 120 | 2.5339 | 30.4068 | 150.4068 |
| 7 | 140 | 2.5086 | 30.1032 | 170.1032 |

Total Cost = 20\*m + 12\*Ls.

So, he should hire 4 mechanics.

Note that in the previous example, waiting cost is assessed only on the “waiting-in-queue” part when planes are airborne, so we use Lq in the calculation of waiting cost. Here the waiting cost is assessed on the total time, so we use Ls, not Lq.***Example 7:*** Cascade Plastics has a large group of molding machines that breakdown at a (Poisson distributed) mean rate of six per (8-hour) day. Each maintenance technician can service an average of one machine per hour (service times are exponentially distributed). Each hour that a machine is down costs the company approximately $50 in lost profit. If maintenance technicians are paid $15 per hour, what size maintenance crew should be hired?

***Solution:***

**This problem is similar to the previous one. So if you feel comfortable you can skip this one.**

This is an M/M/m system where the number of technicians, *m*, needs to be determined. λ = 0.75 machines per hour, μ = 1 machine per hour. We use the “MMm” spreadsheet and set cells E2-E3 to be 0.75 and 1. Then we vary cell E4 (first column in the following table):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **m** | **Cost of mechanics** | **Ls** | **Waiting cost** | **Total cost** |
| 1 | 15 | 3 | 150 | 165 |
| **2** | **30** | **0.8727** | **43.635** | **73.635** |
| 3 | 45 | 0.7647 | 38.235 | 83.235 |
| 4 | 60 | 0.7518 | 37.59 | 97.59 |
| 5 | 75 | 0.7502 | 37.51 | 112.51 |

Cost of service = $15 per technician per hour (this is used to calculate cost of mechanics).

Cost of waiting = $50 per hour per machine down (this is used to calculate waiting cost).

Total Cost = 15\*m + 50\*Ls.

So, he should hire **2** technicians.

***Example 8:*** A beverage store has determined that it is economically feasible to add a drive-in window, with space for two cars: one at the window and one waiting. The owner wants to know whether more waiting space should be leased.

Cars arrive according to a Poisson distribution at a rate of eight per hour. Transactions average 10 per hour, and the times are exponentially distributed. Each transaction makes $1 profit. The owner plans to be open 12 hours per day, 6 days a week, 52 weeks a year. Additional spaces cost $2000 per year to lease. How many spaces is it worthwhile to lease?

***Solution:***

This is an M/M/1/k model with  = 8/hr.,  = 10/hr. and  = 0.8. We are asked to find k.

Other data:

 Cost of Leasing a space = $2,000 per yr. = $0.534 per hr.

 Lost profit / transaction = $1.

Now:

 TC(k) = average hourly total cost when k spaces are leased

 = 0.534 k + 1\* hourly rate of lost customers.

We use the “MMmk-finite queue length” worksheet to find the hourly rate of lost customers. First, we set The results are summarized below.

 **k rate of customers lost (cell F17) TC(k)**

 2 2.098 3.166

 **3 1.388 2.99**

 4 0.975 3.111

As can be seen, it is optimal to lease **3** spaces.