Chapter 7

Confidence Intervals

Chapter Outline

7.1 Large Sample Confidence Intervals for a Population Mean

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*7.5 Confidence Intervals for Parameters of Finite Populations

*7.6 An Introduction to Survey Sampling

*7.7 A Comparison of Confidence Intervals and Tolerance Intervals

*Optional section

We have seen that we obtain a point estimate of a population mean by computing the mean of a sample that has been randomly selected from the population, and we have seen that we obtain a point estimate of a population proportion by computing a sample proportion. In general, we often compute a point estimate of a population parameter (mean, proportion, or the like) in order to make an inference about a population. Unfortunately, a point estimate seldom gives us all the information we need because the point estimate provides no information about its distance from the population parameter.

In this chapter we study how to use a **confidence interval** to estimate a population parameter. A confidence interval for a population parameter is an interval, or range of numbers, constructed around the point estimate so that we are very sure, or confident, that the population parameter is inside the interval. By computing such an interval, we estimate how far the point estimate might be from the parameter.

As we illustrate by revisiting several cases we have introduced in earlier chapters, confidence intervals often help in business decision making and in assessing the need for and the results obtained from process improvement efforts. Specifically,

In the **Car Mileage Case,** we use a confidence interval to provide strong evidence that the mean EPA combined city and highway mileage for the automaker's new midsize model meets the tax credit standard of 31 mpg.

In the **Accounts Receivable Case,** we use a confidence interval to more completely assess the reduction in mean payment time that was achieved by the new billing system.

In the **Marketing Research Case,** we use a confidence interval to provide strong evidence that the mean rating of the new package design exceeds the minimum standard for a successful design.

In the **Cheese Spread Case,** we use a confidence interval to provide strong evidence that fewer than 10 percent of all current purchasers will stop buying the cheese spread if the new spout is used, and, therefore, that it is reasonable to use the new spout.

In the **Marketing Ethics Case,** we use a confidence interval to provide strong evidence that more than half of all marketing researchers disapprove of the actions taken in the ultraviolet ink scenario.

Sections 7.1 through 7.4 present confidence intervals for population means and proportions. These intervals are appropriate when the sampled population is either infinite or finite and *much larger* than (say, at least 20 times as large as) the size of the sample. The appropriate procedures to use when the population is not large compared to the sample are explained in optional Section 7.5. Because not all samples are random samples, optional Section 7.6 briefly discusses three other sampling designs—**stratified, cluster,** and **systematic sampling.** Finally, optional Section 7.7 compares confidence intervals and tolerance intervals.

7.1 Large Sample Confidence Intervals for a Population Mean

An example of calculating and interpreting a confidence interval for μ We have seen that we use the sample mean as the point estimate of the population mean. A **confidence interval** for the population mean is an interval constructed around the sample mean so that we are reasonably sure, or confident, that this interval contains the population mean. For example, suppose in the car mileage case we wish to calculate a confidence interval for the mean μ of the population of all EPA combined city and highway mileages that would be obtained by the new midsize model. To do this, we use the random sample of $n = 49$ mileages given in Table 2.1 (page 40). Before using this sample, however, we first consider calculating a confidence interval for μ by using a smaller sample of $n = 5$ mileages. We do this because it is somewhat simpler to explain the meaning of confidence intervals in terms of smaller samples. Assume, then, that we randomly select a sample of five cars and calculate the mean $\bar{x}$ of the mileages that the cars obtain when tested as prescribed by the EPA. We have seen in Chapter 6 that, if the population of all individual car mileages is normally distributed with mean μ and standard deviation σ, then the sampling distribution of $\bar{x}$ is normal with mean $\mu_{\bar{x}} = \mu$ and standard deviation

CHAPTER 9

TABLE 7.1 The Sample Mean $\bar{x}$ and the Interval $[\bar{x} \pm .7155]$ Given by Each of Three Samples

Sample	Sample	Sample
$x_1 = 30.8$	$x_1 = 32.4$	$x_1 = 32.7$
$x_2 = 31.9$	$x_2 = 30.8$	$x_2 = 31.6$
$x_3 = 30.3$	$x_3 = 31.9$	$x_3 = 33.3$
$x_4 = 32.1$	$x_4 = 31.5$	$x_4 = 32.3$
$x_5 = 31.4$	$x_5 = 32.0$	$x_5 = 32.6$
$\bar{x} = 31.3$	$\bar{x} = 31.72$	$\bar{x} = 32.5$
$[\bar{x} \pm .7155] = [31.3 \pm .7155]$	$[\bar{x} \pm .7155] = [31.72 \pm .7155]$	$[\bar{x} \pm .7155] = [32.5 \pm .7155]$
$= [30.5845, 32.0155]$	$= [31.0045, 32.4355]$	$= [31.7845, 33.2155]$
$= [30.6, 32.0]$	$= [31.0, 32.4]$	$= [31.8, 33.2]$

$\sigma_{\bar{x}} = \sigma/\sqrt{n}$. To obtain a confidence interval for μ, consider using $\bar{x}$ and $\sigma_{\bar{x}}$ to calculate the interval

$$[\bar{x} \pm 2\sigma_{\bar{x}}] = \left[\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}\right] = \left[\bar{x} - 2\frac{\sigma}{\sqrt{5}},\quad \bar{x} + 2\frac{\sigma}{\sqrt{5}}\right]$$

In order to interpret this interval, we find the probability that it contains μ. To do this, we assume that, although we do not know the true value of μ, we do know that the true value of σ is .8 mpg. This is unrealistic, but it simplifies our initial presentation of confidence intervals. Later in this section, we will see that we can calculate a confidence interval without knowing σ. Assuming that σ is .8, we will calculate the interval

$$\left[\bar{x} \pm 2\frac{.8}{\sqrt{5}}\right] = [\bar{x} \pm .7155]$$

Clearly, the interval that we calculate—and whether it contains μ—depends on the sample mean $\bar{x}$ that we obtain. For example, Table 7.1 gives three samples of five mileages that might be randomly selected from the population of all mileages. This table also presents the sample mean $\bar{x}$ and the interval $[\bar{x} \pm .7155]$ that would be given by each sample. In order to see that some intervals contain μ and some do not, suppose that the true value of μ is 31.5 (of course, no human being would know this true value). We then see that the interval given by the leftmost sample in Table 7.1—[30.6, 32.0]—contains μ. Furthermore, the interval given by the middle sample—[31.0, 32.4]—contains μ. However, the interval given by the rightmost sample—[31.8, 33.2]—does not contain μ. Therefore, two out of the three intervals in Table 7.1 contain μ.

In general, there are an infinite number of samples of five mileages that we might randomly select from the population of all mileages. It follows that there are an infinite number of sample means and thus intervals that we might calculate from the samples. To find the probability that such an interval will contain μ, we use the fact that the sampling distribution of the sample mean is normal with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, and we reason as follows:

1 The Empirical Rule for a normally distributed population implies that the probability is .9544 that $\bar{x}$ will be within plus or minus

$$2\sigma_{\bar{x}} = 2\frac{\sigma}{\sqrt{n}} = 2\frac{(.8)}{\sqrt{5}} = .7155$$

of μ. This is illustrated in Figure 7.1.

2 Saying

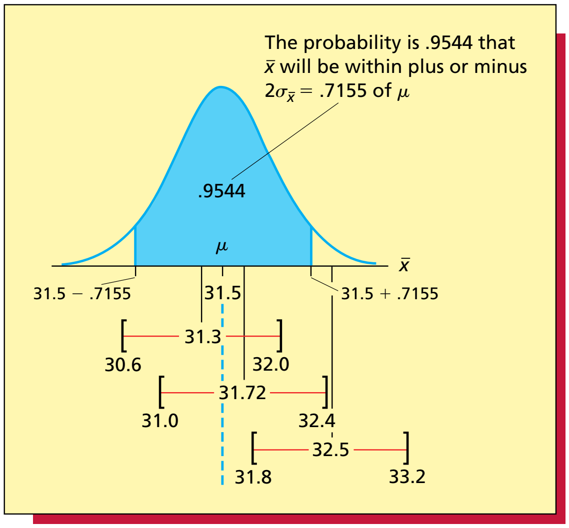
$\bar{x}$ will be within plus or minus .7155 of μ

is the same as saying

$\bar{x}$ will be such that the interval $[\bar{x} \pm .7155]$ contains μ.

To see this, consider Figure 7.1, which illustrates the sample means and intervals given by the three samples in Table 7.1. Note that, because $\bar{x} = 31.3$ is within .7155 of $\mu = 31.5$, the interval $[31.3 \pm .7155] = [30.6, 32.0]$ contains μ. Similarly, since $\bar{x} = 31.72$ is within

FIGURE 7.1 Three 95.44 Percent Confidence Intervals for μ



.7155 of $\mu = 31.5$, the interval $[31.72 \pm .7155] = [31.0, 32.4]$ contains μ. However, because $\bar{x} = 32.5$ is not within .7155 of $\mu = 31.5$, the interval $[32.5 \pm .7155] = [31.8, 33.2]$ does not contain μ.

3 Combining 1 and 2, we see that there is a .9544 probability that $\bar{x}$ will be such that the interval $[\bar{x} \pm .7155]$ contains μ.

Statement 3 says that, **before we randomly select the sample,** there is a .9544 probability that we will obtain an interval $[\bar{x} \pm .7155]$ that contains μ. In other words, 95.44 percent of all intervals that we might obtain contain μ, and 4.56 percent of these intervals do not contain μ. For this reason, we call the interval $[\bar{x} \pm .7155]$ a **95.44 percent confidence interval for μ.** To better understand this interval, we must realize that, **when we actually select the sample,** we will observe one particular sample from the infinite number of possible samples. Therefore, we will obtain one particular confidence interval from the infinite number of possible confidence intervals. For example, suppose that when we actually select the sample of five cars and record their mileages, we obtain the leftmost sample of mileages in Table 7.1. Since the mean of our sample is $\bar{x} = 31.3$, the 95.44 percent confidence interval for μ that it gives is

$$\begin{aligned}[\bar{x} \pm .7155] &= [31.3 \pm .7155] \\ &= [30.6, 32.0]\end{aligned}$$

Because we do not know the true value of μ, we do not know for sure whether μ is contained in our interval. However, we are 95.44 percent confident that μ is contained in this interval. That is, we are 95.44 percent confident that μ is between 30.6 mpg and 32.0 mpg. What we mean by this is that we hope that the interval [30.6, 32.0] is one of the 95.44 percent of all intervals that contain μ and not one of the 4.56 percent of all intervals that do not contain μ.

A general confidence interval formula Later in this section we will see how to make practical use of confidence intervals. First, however, we present a general formula for finding a confidence interval. To do this, recall from the previous example that, before we randomly select the sample, the probability that the confidence interval

$$[\bar{x} \pm 2\sigma_{\bar{x}}] = \left[\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}\right]$$

will contain the population mean is .9544. It follows that the probability that this confidence interval will not contain the population mean is .0456. In general, we denote the probability that a confidence interval for a population mean will *not* contain the population mean by the symbol **α (pronounced alpha).** This implies that **$(1 - \alpha)$, which we call the confidence coefficient,** is

FIGURE 7.2 The Point $z_{\alpha/2}$

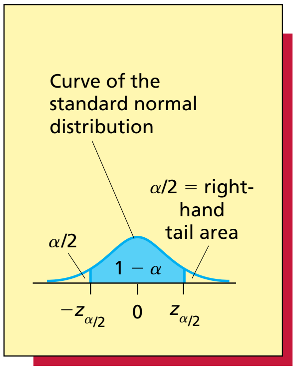


FIGURE 7.3 The Point $z_{.025}$

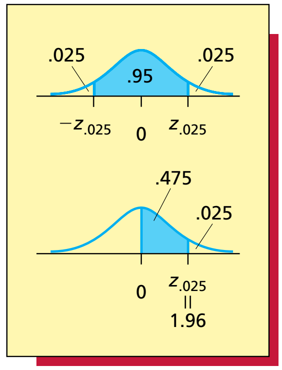
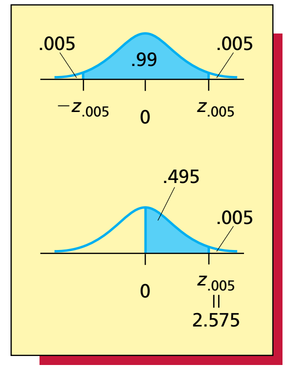


FIGURE 7.4 The Point $z_{.005}$



the probability that the confidence interval will contain the population mean. We can base a confidence interval for a population mean on any confidence coefficient $(1 - \alpha)$ less than 1. However, in practice, we usually use two decimal point confidence coefficients, such as .95 or .99. To find a general formula for a confidence interval for a population mean μ, we assume that the sampled population is normally distributed, or the sample size n is large. Under these conditions, the sampling distribution of the sample mean $\bar{x}$ is exactly (or approximately, by the Central Limit Theorem) a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Then, in order to obtain a confidence interval that has a $(1 - \alpha)$ probability of containing μ, we divide the probability α that the confidence interval will not contain μ into two equal parts, and we place the area $\alpha/2$ in the right-hand tail of the standard normal curve and the area $\alpha/2$ in the left-hand tail of the standard normal curve (see Figure 7.2). We next use the normal table (see Table A.3, page 814) to find the normal point $z_{\alpha/2}$, which is the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to $\alpha/2$. As illustrated in Figure 7.2, the area under the standard normal curve to the right of $z_{\alpha/2}$ is $\alpha/2$, and (by the symmetry of the standard normal curve) the area under this curve to the left of $-z_{\alpha/2}$ is also $\alpha/2$. Therefore, as shown in Figure 7.2, the area under the standard normal curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is $(1 - \alpha)$. At the end of this section we will show that $(1 - \alpha)$ is the probability that the confidence interval

$$[\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}}] = \left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

will contain the population mean μ. In other words, this confidence interval is based on a confidence coefficient of $(1 - \alpha)$, and hence we call this interval a **100(1 − α) percent confidence interval for the population mean.** Here, **100(1 − α) percent** is called the **confidence level** associated with the confidence interval. This *confidence level* is the percentage of time that the confidence interval would contain the population mean if all possible samples were used to calculate this interval.

For example, suppose we wish to find a 95 percent confidence interval for the population mean. Since the confidence level is 95 percent, we have $100(1 - \alpha) = 95$. This implies that the confidence coefficient is $(1 - \alpha) = .95$, which implies that $\alpha = .05$ and $\alpha/2 = .025$. Therefore, we need to find the normal point $z_{.025}$. As shown in Figure 7.3, the area under the standard normal curve between $-z_{.025}$ and $z_{.025}$ is .95, and the area under this curve between 0 and $z_{.025}$ is .475. Looking up the area .475 in Table A.3, we find that $z_{.025} = 1.96$. It follows that the interval

$$[\bar{x} \pm z_{.025}\sigma_{\bar{x}}] = \left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$$

is a 95 percent confidence interval for the population mean μ. This means that if all possible samples were used to calculate this interval, 95 percent of the resulting intervals would contain μ.

As another example, consider a 99 percent confidence interval for the population mean. Because the confidence level is 99 percent, we have $100(1 - \alpha) = 99$, and the confidence coefficient is $(1 - \alpha) = .99$. This implies that $\alpha = .01$ and $\alpha/2 = .005$. Therefore, we need to find the normal point $z_{.005}$. As shown in Figure 7.4, the area under the standard normal curve between

TABLE 7.2 The Normal Point $z_{\alpha/2}$ for Various Levels of Confidence

$100(1 - \alpha)$ percent	α	$\alpha/2$	Normal Point $z_{\alpha/2}$
90% = 100(1 − .10)%	.10	.05	$z_{.05} = 1.645$
95% = 100(1 − .05)%	.05	.025	$z_{.025} = 1.96$
98% = 100(1 − .02)%	.02	.01	$z_{.01} = 2.33$
99% = 100(1 − .01)%	.01	.005	$z_{.005} = 2.575$

$-z_{.005}$ and $z_{.005}$ is .99, and the area under this curve between 0 and $z_{.005}$ is .495. Looking up the area .495 in Table A.3, we find that $z_{.005} = 2.575$. It follows that the interval

$$[\bar{x} \pm z_{.005}\sigma_{\bar{x}}] = \left[\bar{x} \pm 2.575\frac{\sigma}{\sqrt{n}}\right]$$

is a 99 percent confidence interval for the population mean μ. This means that if all possible samples were used to calculate this interval, 99 percent of the resulting intervals would contain μ.

To compare the 95 percent confidence interval with the 99 percent confidence interval, note that the longer of these intervals—$[\bar{x} \pm 2.575(\sigma/\sqrt{n})]$—has an intuitively higher probability (.99) of containing μ. In general, increasing the confidence level (1) has the advantage of making us more confident that μ is contained in the confidence interval, but (2) has the disadvantage of increasing the length of the confidence interval and thus providing a less precise estimate of the true value of μ. Frequently, 95 percent confidence intervals are used to make conclusions. If conclusions based on stronger evidence are desired, 99 percent intervals are sometimes used.

Table 7.2 shows the confidence levels 95 percent and 99 percent, as well as two other confidence levels—90 percent and 98 percent—that are sometimes used to calculate confidence intervals. In addition, this table gives the values of α, $\alpha/2$, and $z_{\alpha/2}$ that correspond to these confidence levels. The following box summarizes the formula used in calculating a $100(1 - \alpha)$ percent confidence interval for a population mean μ. This box also shows what should be done in the likely case that we do not know the true value of the population standard deviation σ.

A Confidence Interval for a Population Mean μ

Suppose that the sampled population is normally distributed or the sample size n is large. Then a **$100(1 - \alpha)$ percent confidence interval for μ** is

$$\left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

If σ is unknown and n is *large (at least 30)*, we estimate σ by the sample standard deviation s. In this case, a $100(1 - \alpha)$ percent confidence interval for μ is

$$\left[\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}\right]$$

Although the first interval in the above box is the theoretically correct confidence interval for μ, we rarely know the true value of σ. Statistical theory and practice tell us that n must be large (at least 30) for s to be an accurate estimate of σ. Therefore, we should not use the second interval in the above box unless n is large. For this reason, we sometimes call this interval a **large sample confidence interval for μ.** In the next section we will see what to do if σ is unknown and the sample size is small.

EXAMPLE 7.1 The Car Mileage Case

Recall that the federal government will give a tax credit to any automaker selling a midsize model equipped with an automatic transmission that has an EPA combined city and highway mileage estimate of at least 31 mpg. Furthermore, to ensure that it does not overestimate a car model's mileage, the EPA will obtain the model's mileage estimate by rounding down—to the nearest mile per gallon—the lower limit of a 95 percent confidence interval for the model's mean mileage μ. That is, the model's mileage estimate is an estimate of the smallest that μ might reasonably be. Suppose the automaker described in Chapter 1 conducts mileage tests on a sample of 49 of its new midsize cars and obtains the sample of 49 mileages in Table 2.1. This sample has mean $\bar{x} = 31.5531$ and standard deviation $s = .7992$. As illustrated in Figure 7.3, in order to compute a 95 percent interval we use the normal point $z_{\alpha/2} = z_{.05/2} = z_{.025} = 1.96$. It follows that the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\right] = \left[31.5531 \pm 1.96\frac{.7992}{\sqrt{49}}\right]$$
$$= [31.5531 \pm .2238]$$
$$= [31.33, 31.78]$$

This interval says we are 95 percent confident that the model's mean mileage μ is between 31.33 mpg and 31.78 mpg. Based on this interval, the model's EPA mileage estimate is 31 mpg, and the automaker will receive the tax credit.

If we wish to compute a 99 percent confidence interval for μ, then, as illustrated in Figure 7.4, we use the normal point $z_{\alpha/2} = z_{.01/2} = z_{.005} = 2.575$. It follows that the 99 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.005}\frac{s}{\sqrt{n}}\right] = \left[31.5531 \pm 2.575\frac{.7992}{\sqrt{49}}\right]$$
$$= [31.5531 \pm .2940]$$
$$= [31.26, 31.85]$$

This interval says we are 99 percent confident that the model's mean mileage μ is between 31.26 mpg and 31.85 mpg. Notice that, although the 99 percent interval is slightly longer that the 95 percent interval, there is little difference between the lengths of these intervals. The reason is that the large sample size, $n = 49$, eliminates most of the difference between the lengths of the intervals.

EXAMPLE 7.2 The Accounts Receivable Case

Recall that a management consulting firm has installed a new computer-based, electronic billing system in a Hamilton, Ohio, trucking company. The mean payment time using the trucking company's old billing system was approximately equal to, but no less than, 39 days. In order to assess whether the mean payment time, μ, using the new billing system is substantially less than 39 days, the consulting firm will use the sample of $n = 65$ payment times in Table 1.2 to find a 95 percent confidence interval for μ. The mean and the standard deviation of the 65 payment times are $\bar{x} = 18.1077$ and $s = 3.9612$. Using the normal point $z_{\alpha/2} = z_{.025} = 1.96$, it follows that the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\right] = \left[18.1077 \pm 1.96\frac{3.9612}{\sqrt{65}}\right]$$
$$= [18.1077 \pm .9630]$$
$$= [17.1, 19.1]$$

Recalling that the mean payment time using the old billing system is 39 days, the point estimate $\bar{x} = 18.1$ says we estimate that the new billing system reduces the mean payment time by 20.9 days. Because the interval says that we are 95 percent confident that the mean payment time using the new billing system is between 17.1 days and 19.1 days, we are 95 percent confident that the new billing system reduces the mean payment time by at most 21.9 days and by at least 19.9 days.

EXAMPLE 7.3 The Marketing Research Case

Recall that a brand group is considering a new package design for a line of cookies and that Table 1.3 gives a random sample of $n = 60$ consumer ratings of this new package design. Let μ denote the mean rating of the new package design that would be given by all consumers. In order to assess whether μ exceeds the minimum standard composite score of 25 for a successful package design, the brand group will calculate a 95 percent confidence interval for μ. The mean and the standard deviation of the package design ratings are $\bar{x} = 30.35$ and $s = 3.1073$. Using the normal point $z_{\alpha/2} = z_{.025} = 1.96$, the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\right] = \left[30.35 \pm 1.96\frac{3.1073}{\sqrt{60}}\right]$$
$$= [30.35 \pm .7863]$$
$$= [29.6, 31.1]$$

The point estimate $\bar{x} = 30.35$ says we estimate that the mean rating of the new package design is 5.35 points higher than the minimum standard of 25 for a successful package design. Since the interval says we are 95 percent confident that the mean rating of the new package design is between 29.6 and 31.1, we are 95 percent confident that this mean rating exceeds the minimum standard of 25 by at least 4.6 points and by at most 6.1 points.

Justifying the confidence interval formula To show why the interval

$$\left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

is a $100(1 - \alpha)$ percent confidence interval for μ, recall that if the sampled population is normally distributed or the sample size n is large, then the sampling distribution of $\bar{x}$ is (exactly or approximately) a normal distribution with mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. It follows that the sampling distribution of

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

is (exactly or approximately) a standard normal distribution. Therefore, the probability that we will obtain a sample mean $\bar{x}$ such that z is between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is $1 - \alpha$ (see Figure 7.5). That is, we can say that the probability that

$$-z_{\alpha/2} \le \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \le z_{\alpha/2}$$

equals $1 - \alpha$. Using some algebraic manipulations, we can show that this is equivalent to saying that the probability that

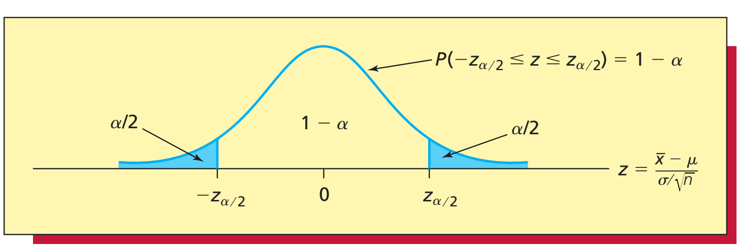
$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

equals $1 - \alpha$. This probability statement says that the probability is $1 - \alpha$ (for example, .95) that we will obtain a sample mean $\bar{x}$ such that the interval

$$\left[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

contains μ. In other words, this interval is a $100(1 - \alpha)$ percent confidence interval for μ.

FIGURE 7.5 A Probability for Deriving a Confidence Interval for the Population Mean



Exercises for Section 7.1

7.4, 7.5, 7.7, 7.8

CONCEPTS

7.1 Explain why it is important to calculate a confidence interval in addition to calculating a point estimate of a population parameter.

7.2 Write a paragraph explaining exactly what the term "95 percent confidence" means in the context of calculating a 95 percent confidence interval for a population mean.

7.3 For each of the following changes, indicate whether a confidence interval for μ will become longer or shorter:

a An increase in the level of confidence.
b An increase in the sample size.
c A decrease in the level of confidence.
d A decrease in the sample size.

METHODS AND APPLICATIONS

7.4 For each of the following confidence levels, $100(1 - \alpha)$ percent, find the $z_{\alpha/2}$ point needed to compute a confidence interval for μ:

a 95% **c** 99.73% **e** 97%
b 99% **d** 80% **f** 92%

7.5 Suppose that, for a sample of size $n = 100$ measurements, we find that $\bar{x} = 50$ and $s = 2$. Calculate confidence intervals for the population mean μ with the following confidence levels:

a 95% **b** 99% **c** 97% **d** 80% **e** 99.73%

7.6 THE TRASH BAG CASE TrashBag

Consider the trash bag problem. Suppose that an independent laboratory has tested trash bags and has found that no 30-gallon bags that are currently on the market have a mean breaking strength of 50 pounds or more. On the basis of these results, the producer of the new, improved trash bag feels sure that its 30-gallon bag will be the strongest such bag on the market if the new trash bag's mean breaking strength can be shown to be at least 50 pounds. The mean and the standard deviation of the sample of 40 trash bag breaking strengths in Table 1.10 are $\bar{x} = 50.575$ and $s = 1.6438$. If we let μ denote the mean of the breaking strengths of all possible trash bags of the new type,

a Calculate 95 percent and 99 percent confidence intervals for μ.
b Using the 95 percent confidence interval, can we be 95 percent confident that μ is at least 50 pounds? Explain.
c Using the 99 percent confidence interval, can we be 99 percent confident that μ is at least 50 pounds? Explain.
d Based on your answers to parts *b* and *c*, how convinced are you that the new 30-gallon trash bag is the strongest such bag on the market?

7.7 THE BANK CUSTOMER WAITING TIME CASE WaitTime

Recall that a bank manager has developed a new system to reduce the time customers spend waiting to be served by tellers during peak business hours. The mean waiting time during peak business hours under the current system is roughly 9 to 10 minutes. The bank manager hopes that the new system will have a mean waiting time that is less than six minutes. The mean and the standard deviation of the sample of 100 bank customer waiting times in Table 1.6 are $\bar{x} = 5.46$ and $s = 2.475$. If we let μ denote the mean of all possible bank customer waiting times using the new system,

a Calculate 95 percent and 99 percent confidence intervals for μ.
b Using the 95 percent confidence interval, can the bank manager be 95 percent confident that μ is less than six minutes? Explain.
c Using the 99 percent confidence interval, can the bank manager be 99 percent confident that μ is less than six minutes? Explain.
d Based on your answers to parts *b* and *c*, how convinced are you that the new mean waiting time is less than six minutes?

7.8 THE CUSTOMER SATISFACTION RATING CASE CustSat

The mean and the standard deviation of the sample of 65 customer satisfaction ratings in Table 1.5 are $\bar{x} = 42.95$ and $s = 2.6424$. If we let μ denote the mean of all possible customer satisfaction ratings for the VAC-5000,

a Calculate 95 percent and 99 percent confidence intervals for μ.
b Using the 95 percent confidence interval, can we be 95 percent confident that μ is greater than 42 (recall that a very satisfied customer gives a rating greater than 42)? Explain.

c Using the 99 percent confidence interval, can we be 99 percent confident that μ is greater than 42? Explain.

d Based on your answers to parts *b* and *c*, how convinced are you that the mean satisfaction rating indicates that a typical customer is very satisfied?

7.9 In an article in the *Journal of Management,* Morris, Avila, and Allen studied innovation by surveying firms to find (among other things) the number of new products introduced by the firms. Suppose a random sample of 100 California-based firms is selected and each firm is asked to report the number of new products it has introduced during the last year. The sample mean is found to be $\bar{x} = 5.68$ and the sample standard deviation is $s = 8.70$.

a Calculate a 98 percent confidence interval for the population mean number of new products introduced in the last year.

b Based on your confidence interval, find a reasonable estimate for the smallest value that the mean number of new products might be. Explain.

7.10 In an article in *Marketing Science,* Silk and Berndt investigate the output of advertising agencies. They describe ad agency output by finding the shares of dollar billing volume coming from various media categories such as network television, spot television, newspapers, radio, and so forth.

a Suppose that a random sample of 400 U.S. advertising agencies gives an average percentage share of billing volume from network television equal to 7.46 percent with a standard deviation of 1.42 percent. Calculate a 95 percent confidence interval for the mean percentage share of billing volume from network television for the population of all U.S. advertising agencies.

b Suppose that a random sample of 400 U.S. advertising agencies gives an average percentage share of billing volume from spot television commercials equal to 12.44 percent with a standard deviation of 1.55 percent. Calculate a 95 percent confidence interval for the mean percentage share of billing volume from spot television commercials for the population of all U.S. advertising agencies.

c Compare the confidence intervals in parts *a* and *b*. Does it appear that the mean percentage share of billing volume from spot television commercials for U.S. advertising agencies is greater than the mean percentage share of billing volume from network television? Explain.

7.11 In an article in *Accounting and Business Research,* Carslaw and Kaplan investigate factors that influence "audit delay" for firms in New Zealand. Audit delay, which is defined to be the length of time (in days) from a company's financial year-end to the date of the auditor's report, has been found to affect the market reaction to the report. This is because late reports seem to often be associated with lower returns and early reports seem to often be associated with higher returns.

Carslaw and Kaplan investigated audit delay for two kinds of public companies—owner-controlled and manager-controlled companies. Here a company is considered to be owner controlled if 30 percent or more of the common stock is controlled by a single outside investor (an investor not part of the management group or board of directors). Otherwise, a company is considered manager controlled. It was felt that the type of control influences audit delay. To quote Carslaw and Kaplan:

> Large external investors, having an acute need for timely information, may be expected to pressure the company and auditor to start and to complete the audit as rapidly as practicable.

a Suppose that a random sample of 100 public owner-controlled companies in New Zealand is found to give a mean audit delay of $\bar{x} = 82.6$ days with a standard deviation of $s = 32.83$ days. Calculate a 95 percent confidence interval for the population mean audit delay for all public owner-controlled companies in New Zealand.

b Suppose that a random sample of 100 public manager-controlled companies in New Zealand is found to give a mean audit delay of $\bar{x} = 93$ days with a standard deviation of $s = 37.18$ days. Calculate a 95 percent confidence interval for the population mean audit delay for all public manager-controlled companies in New Zealand.

c Use the confidence intervals you computed in parts *a* and *b* to compare the mean audit delay for all public owner-controlled companies versus that of all public manager-controlled companies. How do the means compare? Explain.

7.12 In an article in the *Journal of Marketing,* Bayus studied the differences between "early replacement buyers" and "late replacement buyers" in making consumer durable good replacement purchases. Early replacement buyers are consumers who replace a product during the early part of its lifetime, while late replacement buyers make replacement purchases late in the product's lifetime. In particular, Bayus studied automobile replacement purchases. Consumers who traded in cars with ages of zero to three years and mileages of no more than 35,000 miles were classified as early replacement buyers. Consumers who traded in cars with ages of seven or more years and mileages of more than 73,000 miles were classified as late replacement buyers. Bayus compared the two groups of buyers with respect to demographic variables such as income, education, age, and so forth. He also compared the two groups with respect to the amount of search activity in the replacement purchase process. Variables compared included the number of dealers visited, the time spent gathering information, and the time spent visiting dealers.

a Suppose that a random sample of 800 early replacement buyers yields a mean number of dealers visited of $\bar{x} = 3.3$ with a standard deviation of $s = .71$. Calculate a 99 percent confidence interval for the population mean number of dealers visited by early replacement buyers.

b Suppose that a random sample of 500 late replacement buyers yields a mean number of dealers visited of $\bar{x} = 4.3$ with a standard deviation of $s = .66$. Calculate a 99 percent confidence interval for the population mean number of dealers visited by late replacement buyers.

c Use the confidence intervals you computed in parts *a* and *b* to compare the mean number of dealers visited by early replacement buyers with the mean number of dealers visited by late replacement buyers. How do the means compare? Explain.

7.2 Small Sample Confidence Intervals for a Population Mean

If the population sampled is normally distributed, or if the sample size is large, then the sampling distribution of

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

is (exactly or approximately) a standard normal distribution. Because the value of σ is rarely known, and because the sample size must be large (at least 30) to replace σ by s, the confidence intervals presented in Section 7.1 are most often used when we take a large random sample. If we take a small random sample and do not know σ, then we can construct a confidence interval for μ that is based on the sampling distribution of

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

CHAPTER 5

If the sampled population is normally distributed, then for any sample size n this sampling distribution is what is called a ***t* distribution.**

The curve of the t distribution has a shape similar to that of the standard normal curve. Two t curves and a standard normal curve are illustrated in Figure 7.6. A t curve is symmetrical about zero, which is the mean of any t distribution. However, the t distribution is more spread out, or variable, than the standard normal distribution. Since the above t statistic is a function of two random variables, $\bar{x}$ and s, it is logical that the sampling distribution of this statistic is more variable than the sampling distribution of the z statistic, which is a function of only one random variable, $\bar{x}$. The exact spread, or standard deviation, of the t distribution depends on a parameter that is called the **number of degrees of freedom (denoted *df*).** The degrees of freedom *df* varies depending on the problem. In the present situation the sampling distribution of t has a number of degrees of freedom that equals the sample size minus 1. We say that this sampling distribution is a ***t* distribution with $n - 1$ degrees of freedom.** As the sample size n (and thus the number of degrees of freedom) increases, the spread of the t distribution decreases (see Figure 7.6). Furthermore, as the number of degrees of freedom approaches infinity, the curve of the t distribution approaches (that is, becomes shaped more and more like) the curve of the standard normal distribution. In fact, when the sample size n is at least 30 and thus the number of degrees of freedom $n - 1$ is at least 29, the curve of the t distribution is very similar to the standard normal curve.

In order to use the t distribution, we employ a ***t* point that is denoted t_α.** As illustrated in Figure 7.7, **t_α is the point on the horizontal axis under the curve of the *t* distribution that gives a right-hand tail area equal to α.** The value of t_α in a particular situation depends upon the right-hand tail area α and the number of degrees of freedom (*df*) of the t distribution. Values of t_α are tabulated in a ***t* table.** Such a table is given in Table A.4 of Appendix A (page 815) and

FIGURE 7.6 As the Number of Degrees of Freedom Increases, the Spread of the t Distribution Decreases and the t Curve Approaches the Standard Normal Curve

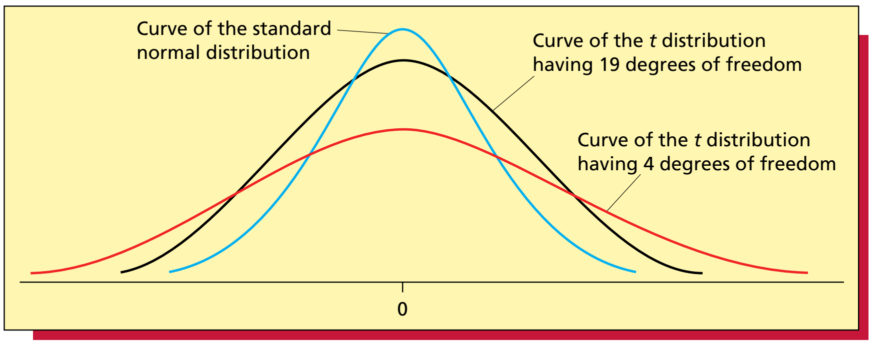


FIGURE 7.7 An Example of a t Point Giving a Specified Right-Hand Tail Area (This t Point Gives a Right-Hand Tail Area Equal to α).

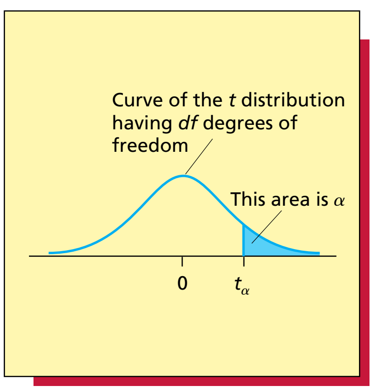
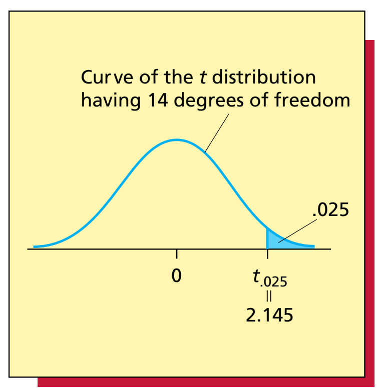


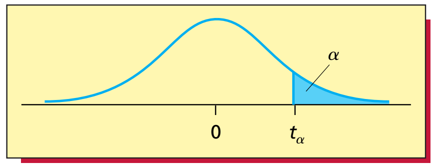
FIGURE 7.8 The t Point Giving a Right-Hand Tail Area of .025 under the t Curve Having 14 Degrees of Freedom: $t_{.025} = 2.145$



is reproduced in this chapter as Table 7.3. In this t table, the rows correspond to the appropriate number of degrees of freedom (values of which are listed down the left side of the table), while the columns designate the right-hand tail area α. For example, suppose we wish to find the t point that gives a right-hand tail area of .025 under a t curve having 14 degrees of freedom. To do this, we look in Table 7.3 at the row labeled 14 and the column labeled $t_{.025}$. We find that this $t_{.025}$ point is 2.145 (also see Figure 7.8). Similarly, when there are 14 degrees of freedom, we find that $t_{.005} = 2.977$ (see Table 7.3 and Figure 7.9).

The t table gives t points for degrees of freedom from 1 to 30. The table also gives t points for 40, 60, 120, and an infinite number of degrees of freedom. Most values of the degrees of freedom greater than 30 are not tabled. This is because, when the number of degrees of freedom is large, the t point is very close to the normal point that gives the same right-hand tail area. In general, if the number of degrees of freedom is at least 29, then it is sufficient to use the normal point for most applications. Furthermore, it is useful to realize that the normal points giving the various right-hand tail areas are listed in the row of the t table corresponding to an infinite (∞) number of degrees of freedom. Looking at the row corresponding to ∞,we see that, for example, $z_{.025} = 1.96$ and $z_{.005} = 2.576$. Therefore, we can use this row in the t table as an alternative to using the normal table when we need to find normal points (such as $z_{\alpha/2}$ in Section 7.1).

TABLE 7.3 A *t* Table



df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: E. S. Pearson and H. O. Hartley eds., *The Biometrika Tables for Statisticians* 1, 3d ed. (Biometrika, 1966). Reproduced by permission of Oxford University Press Biometrika Trustees.

We now present the formula for a $100(1 - \alpha)$ percent confidence interval for a population mean μ based on the *t* distribution.

A Small Sample $100(1 - \alpha)$ percent Confidence Interval for a Population Mean μ

If the sampled population is normally distributed with mean μ, then a **$100(1 - \alpha)$ percent confidence interval for μ** is

$$\left[\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}\right]$$

Here $t_{\alpha/2}$ is the *t* point giving a right-hand tail area of $\alpha/2$ under the *t* curve having $n - 1$ degrees of freedom, and n is the sample size.

CHAPTER 9

FIGURE 7.9 **The t Point Giving a Right-Hand Tail Area of .005 under the t Curve Having 14 Degrees of Freedom: $t_{.005} = 2.977$**

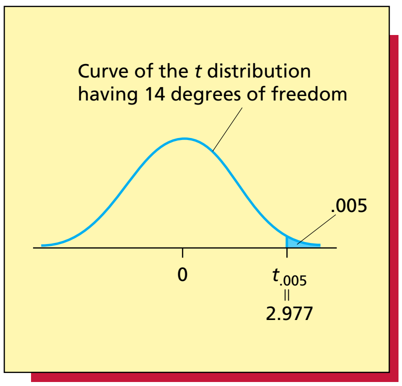
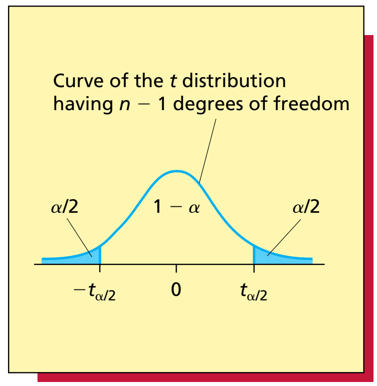


FIGURE 7.10 **The Point $t_{\alpha/2}$ with $n - 1$ Degrees of Freedom**



Before presenting an example, we need to make a few comments. First, it has been shown that this confidence interval is approximately valid for many populations that are not exactly normally distributed. In particular, this interval is approximately valid for a mound-shaped, or single-peaked, population, even if the population is somewhat skewed to the right or left. Second, this small sample interval employs the t point $t_{\alpha/2}$. As shown in Figure 7.10, this t point gives a right-hand tail area equal to $\alpha/2$ under the t curve having $n - 1$ degrees of freedom. Here $\alpha/2$ is determined from the desired confidence level $100(1 - \alpha)$ percent.

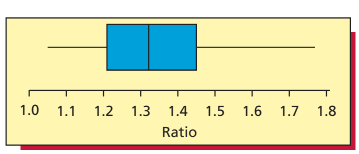
EXAMPLE 7.4

One measure of a company's financial health is its *debt-to-equity ratio.* This quantity is defined to be the ratio of the company's corporate debt to the company's equity. If this ratio is too high, it is one indication of financial instability. For obvious reasons, banks often monitor the financial health of companies to which they have extended commercial loans. Suppose that, in order to reduce risk, a large bank has decided to initiate a policy limiting the mean debt-to-equity ratio for its portfolio of commercial loans to 1.5. In order to estimate the mean debt-to-equity ratio of its loan portfolio, the bank randomly selects a sample of 15 of its commercial loan accounts. Audits of these companies result in the following debt-to-equity ratios:

1.31	1.05	1.45	1.21	1.19
1.78	1.37	1.41	1.22	1.11
1.46	1.33	1.29	1.32	1.65

1.0	5
1.1	1 9
1.2	1 2 9
1.3	1 2 3 7
1.4	1 5 6
1.5	
1.6	5
1.7	8

A stem-and-leaf display of these ratios is given on the page margin, and a box plot of the ratios is given below. The stem-and-leaf display looks reasonably mound-shaped, and both the stem-and-leaf display and the box plot look reasonably symmetrical. Furthermore, the sample mean and standard deviation of the ratios can be calculated to be $\bar{x} = 1.343$ and $s = .192$.



Suppose the bank wishes to calculate a 95 percent confidence interval for the loan portfolio's mean debt-to-equity ratio, μ. Because the bank has taken a sample of size $n = 15$, we have $n - 1 = 15 - 1 = 14$ degrees of freedom, and the level of confidence $100(1 - \alpha)\% = 95\%$

implies that $\alpha = .05$. Therefore, we use the t point $t_{\alpha/2} = t_{.05/2} = t_{.025} = 2.145$ (see Table 7.3). It follows that the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[1.343 \pm 2.145\frac{.192}{\sqrt{15}}\right]$$
$$= [1.343 \pm 0.106]$$
$$= [1.237, 1.449]$$

This interval says the bank is 95 percent confident that the mean debt-to-equity ratio for its portfolio of commercial loan accounts is between 1.237 and 1.449. Based on this interval, the bank has strong evidence that the portfolio's mean ratio is less than 1.5 (or that the bank is in compliance with its new policy).

In general, since the t distribution is more spread out than the standard normal distribution, $t_{\alpha/2}$ is greater than $z_{\alpha/2}$. This can be verified by looking at any column in Table 7.3, and by noting that the smallest entry in the column is the entry in the bottom row (corresponding to an infinite number of degrees of freedom), which is the normal point $z_{\alpha/2}$. For instance, when there are 14 degrees of freedom, $t_{.025} = 2.145$ is greater than $z_{.025} = 1.96$. Now, if we assume that the population sampled is (at least approximately) normally distributed, then the 95 percent confidence interval for μ that we calculate if we do not know σ is (for $n = 15$)

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[\bar{x} \pm 2.145\frac{s}{\sqrt{n}}\right]$$

On the other hand, the 95 percent confidence interval for μ that we calculate if we do know σ is

$$\left[\bar{x} \pm z_{.025}\frac{\sigma}{\sqrt{n}}\right] = \left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$$

Intuitively, we see that there is a penalty for not knowing σ: The multiplier ($t_{.025} = 2.145$) of $s/\sqrt{n}$ in the interval based on t is larger than the multiplier ($z_{.025} = 1.96$) of $\sigma/\sqrt{n}$ in the interval based on z.

We have said in Section 7.1 that if we do not know σ (which is almost always the case) and the sample size n is large (say, at least 30), then we can estimate σ by s in the z-based confidence interval for μ and use the large sample, z-based confidence interval. It can be argued, however, that because the t-based confidence interval is a statistically correct interval that does not require that we know σ, then it is best, if we do not know σ, to use this interval for any size sample—even for a large sample. Most common t tables give t points for degrees of freedom from 1 to 30, so we would need a more complete t table or a computer software package to use the t-based confidence interval for a sample whose size n exceeds 31. For example, Figure 7.11(a) gives the Excel output of the sample mean $\bar{x} = 31.5531$ (see "Mean") and the t-based 95 percent confidence interval **half-length** $t_{.025}(s/\sqrt{n}) = .22957$ (see "Confidence Level (95.0%)") in the car mileage case. Here $n = 49$, and thus $t_{.025}$ is based on 48 degrees of freedom. It follows that a t-based 95 percent confidence interval for the mean mileage, μ, of the new midsize model is $[31.5531 \pm .2296] = [31.3235, 31.7827]$. This interval is slightly longer than the large sample, z-based 95 percent confidence interval for μ, which has been calculated in Example 7.1 to be $[31.5531 \pm .2238] = [31.3293, 31.7769]$. In general, when the sample size n is large (say, at least 30), $t_{\alpha/2}$ (based on $n - 1$ degrees of freedom) is slightly larger than $z_{\alpha/2}$, and thus the t-based $100(1 - \alpha)$ percent confidence interval for μ is slightly longer than the large sample, z-based $100(1 - \alpha)$ percent confidence interval for μ. However, because these intervals do not differ by much when n is at least 30, it is reasonable, if n is at least 30, to use the large sample, z-based interval as an approximation to the t-based interval. Furthermore, the Central Limit Theorem tells us that, if n is at least 30, then both intervals are at least approximately valid no matter how the sampled population is distributed. Of course, if the sample size n is small (less than 30) and we do not know σ, then we must use the t-based interval, which is valid for small samples if the sampled population is normally distributed (or at least mound-shaped). Figure 7.11(b) gives the Excel output of the sample mean and the t-based 95 percent confidence interval half-length computed using the sample of $n = 15$ debt-to-equity ratios in Example 7.4. Figure 7.12(a) gives the MINITAB output of the t-based 95 percent confidence interval for the mean debt-to-equity ratio,

FIGURE 7.11 Excel and MINITAB Outputs for the Car Mileage Case, the Debt-to-Equity Ratio Example, and the Accounts Receivable Case

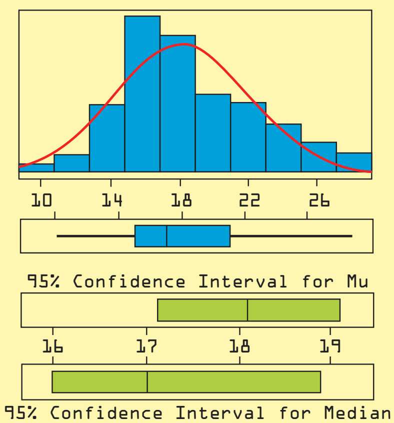
(a) Excel output in the car mileage case

STATISTICS	
Mean	31.55306
Standard Error	0.114178
Median	31.6
Mode	31.7
Standard Deviation	0.799245
Sample Variance	0.638793
Kurtosis	−0.57098
Skewness	−0.10602
Range	3.5
Minimum	29.8
Maximum	33.3
Sum	1546.1
Count	49
Confidence Level(95.0%)	0.22957

(b) Excel output in the debt-to-equity ratio example

STATISTICS	
Mean	1.343333
Standard Error	0.049595
Median	1.32
Mode	#N/A
Standard Deviation	0.192081
Sample Variance	0.036895
Kurtosis	0.833414
Skewness	0.805013
Range	0.73
Minimum	1.05
Maximum	1.78
Sum	20.15
Count	15
Confidence Level(95.0%)	0.106371

(c) MINITAB output in the accounts receivable case

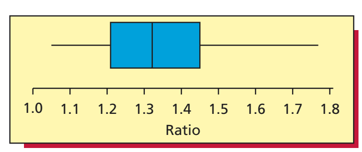



```
Variable: PayTime

Anderson-Darling Normality Test

A-Squared:              0.918
P-Value:                0.018

Mean                  18.1077
StDev                  3.9612
Variance              15.6913
Skewness             0.600799
Kurtosis            3.38E-02
N                          65

Minimum               10.0000
1st Quartile          15.0000
Median                17.0000
3rd Quartile          21.0000
Maximum               29.0000
95% Confidence Interval for Mu
   17.1261            19.0892
95% Confidence Interval for Sigma
    3.3781             4.7896
95% Confidence Interval for Median
   16.0000            18.8785
```

FIGURE 7.12 MINITAB and MegaStat Outputs of a *t*-Based 95 Percent Confidence Interval for the Mean Debt-to-Equity Ratio

(a) The MINITAB output

```
Variable   N     Mean     StDev   SE Mean        95.0 % CI
Ratio     15   1.3433    0.1921    0.0496  ( 1.2370, 1.4497)
```



(b) The MegaStat output

Confidence interval - mean

1.3433 mean	15 n	**1.4497 upper confidence limit**
0.1921 std. dev	2.145 t (df = 14)	**1.2369 lower confidence limit**

which (as hand calculated in Example 7.4) is [1.2369, 1.4497]. The MINITAB output also gives the sample mean $\bar{x}$, as well as the sample standard deviation s and the quantity $s/\sqrt{n}$, which is called the **standard error of the estimate** $\bar{x}$ and denoted "SE Mean" on the MINITAB output (note that $s/\sqrt{n}$ is given on the Excel output as "Standard Error"). Finally, the MINITAB output gives a box plot of the sample of 15 debt-to-equity ratios and graphically illustrates under the box plot the 95 percent confidence interval for the mean debt-to-equity ratio. Figure 7.12(b) gives the MegaStat output of the 95 percent confidence interval for the mean debt-to-equity ratio.

To conclude this section, we note that if the sample size n is small and the sampled population is not mound-shaped or is highly skewed, then the t-based confidence interval for the population mean might not be valid. In this case we can use a **nonparametric method**—a method that makes no assumption about the shape of the sampled population and is valid for any sample size—to find a confidence interval for the **population median.** For example, Figure 7.11(c) gives the MINITAB output of a 95 percent confidence interval for the population median payment time using the new electronic billing system in the accounts receivable case. This figure also shows the 95 percent t-based confidence interval for the population mean and a 95 percent confidence interval for the population standard deviation (in Section 8.8 we discuss how this latter interval is calculated). Because the histogram of payment times is mound-shaped and not highly skewed to the right, both the population mean and the population median are reasonable measures of central tendency. The confidence interval for the population median is particularly useful if the sampled population is highly skewed and is valid even if the sample size is small. In Chapter 15 we further discuss nonparametric methods.

Exercises for Section 7.2

CONCEPTS

7.15, 7.17, 7.20, 7.21

7.13 Explain how each of the following changes as *the number of degrees of freedom* describing a t curve *increases:*

a The standard deviation of the t curve. **b** The points t_α and $t_{\alpha/2}$.

7.14 Discuss when it is appropriate to use the t-based confidence interval for μ.

METHODS AND APPLICATIONS

7.15 Using Table 7.3, find $t_{.10}$, $t_{.025}$, and $t_{.001}$ based on 11 degrees of freedom.

7.16 Using Table 7.3, find $t_{.05}$, $t_{.005}$, and $t_{.0005}$ based on 6 degrees of freedom.

7.17 Suppose that for a sample of $n = 11$ measurements, we find that $\bar{x} = 72$ and $s = 5$. Assuming normality, compute confidence intervals for the population mean μ with the following levels of confidence:

a 95% **b** 99% **c** 80% **d** 90% **e** 98% **f** 99.8%

7.18 The *bad debt ratio* for a financial institution is defined to be the dollar value of loans defaulted divided by the total dollar value of all loans made. Suppose a random sample of seven Ohio banks is selected and that the bad debt ratios (written as percentages) for these banks are 7 percent, 4 percent, 6 percent, 7 percent, 5 percent, 4 percent, and 9 percent. Assuming the bad debt ratios are approximately normally distributed, the MINITAB output of a 95 percent confidence interval for the mean bad debt ratio of all Ohio banks is as follows:

```
Variable   N    Mean   StDev   SE Mean      95.0 % CI
d-ratio    7   6.000   1.826     0.690   ( 4.311,   7.689)
```

a Using the $\bar{x}$ and s on the MINITAB output, verify the calculation of the 95 percent confidence interval, and calculate a 99 percent confidence interval for the mean debt-to-equity ratio.

b Banking officials claim the mean bad debt ratio for all banks in the Midwest region is 3.5 percent and that the mean bad debt ratio for Ohio banks is higher. Using the 95 percent confidence interval, can we be 95 percent confident that this claim is true? Using the 99 percent confidence interval, can we be 99 percent confident that this claim is true?

7.19 In an article in *Quality Progress,* Blauw and During study how long it takes Dutch companies to complete five stages in the adoption of total quality control (TQC). According to Blauw and During, the adoption of TQC can be divided into five stages as follows: TQC

1 Knowledge: the organization has heard of TQC.
2 Attitude formation: the organization seeks information and compares advantages and disadvantages.
3 Decision making: the organization decides to implement TQC.
4 Implementation: the organization implements TQC.
5 Confirmation: the organization decides to apply TQC as a normal business activity.

Suppose a random sample of five Dutch firms that have adopted TQC is selected. Each firm is asked to report how long it took to complete the implementation stage. The firms report the following durations (in years) for this stage: 2.5, 1.5, 1.25, 3.5, and 1.25. Assuming that the durations are approximately normally distributed, the MegaStat output of a 95 percent confidence interval for the mean duration of the implementation stage for Dutch firms is as follows:

Confidence interval - mean

2 mean	5 n	**3.222 upper confidence limit**
0.984 std. dev.	2.776 t (df = 4)	**0.778 lower confidence limit**

Based on the 95 percent confidence interval, is there conclusive evidence that the mean duration of the implementation stage exceeds one year? Explain. What is one possible reason for the lack of conclusive evidence?

7.20 A federal agency wishes to assess the effectiveness of a new air traffic control display panel. The mean time required for air traffic controllers to stabilize an air traffic emergency in which two aircraft have been assigned to the same air space is known to be roughly equal to, but no less than, 17 seconds when the current display panel is used. In order to test the new display panel, 20 air traffic controllers are randomly selected and each is trained to use the new panel. When each randomly selected controller uses the new display panel to stabilize a simulated emergency in which two aircraft have been assigned to the same air space, the mean and standard deviation of the 20 stabilization times so obtained are $\bar{x} = 13.8$ seconds and $s = 1.57$ seconds.

a Assuming that stabilization times are approximately normally distributed, find a 95 percent confidence interval for the true mean time required to stabilize the emergency situation using the new display panel.

b Are we 95 percent confident that the mean stabilization time using the new display panel is less than the 17 seconds for the current display panel? Explain.

7.21 In its October 7, 1991, issue, *Fortune* magazine reported on the rapid rise of fees and expenses charged by various types of mutual funds. As stated in the article:

> Ten years ago the stock funds tracked by Morningstar, a Chicago rating service, took out an average of 1.19% annually. Today they extract 1.59%.

a Suppose the average annual expense for a random sample of 12 stock funds is 1.63 percent with a standard deviation of .31 percent. Calculate a 95 percent confidence interval for the mean annual expense charged by all stock funds. Assume that stock fund annual expenses are approximately normally distributed.

b Suppose the average annual expense for a random sample of 12 municipal bond funds is 0.89 percent with a standard deviation of .23 percent. Calculate a 95 percent confidence interval for the mean annual expense charged by all municipal bond funds. Assume that municipal bond fund expenses are approximately normally distributed.

c Use the 95 percent confidence intervals you computed in parts *a* and *b* to compare the average annual expense for stock funds with that for municipal bond funds. How do the averages compare? Explain.

7.22 A production supervisor at a major chemical company wishes to determine whether a new catalyst, catalyst XA-100, increases the mean hourly yield of a chemical process beyond the current mean hourly yield, which is known to be roughly equal to, but no more than, 750 pounds per hour. To test the new catalyst, five trial runs using catalyst XA-100 are made. The resulting yields for the trial runs (in pounds per hour) are 801, 814, 784, 836, and 820. Assuming that all factors affecting yields of the process have been held as constant as possible during the test runs, it is reasonable to regard the five yields obtained using the new catalyst as a random sample from the population of all possible yields that would be obtained by using the new catalyst. Furthermore, we will assume that this population is approximately normally distributed. ChemYield

a Using the Excel output in Figure 7.13, find a 95 percent confidence interval for the mean of all possible yields obtained using catalyst XA-100.

b Based on the confidence interval, can we be 95 percent confident that the mean yield using catalyst XA-100 exceeds 750 pounds per hour? Explain.

7.23 THE TRASH BAG CASE TrashBag

Using the Excel output in Figure 7.14, find a t-based 95 percent confidence interval for the mean of the breaking strengths of all possible trash bags. Compare this interval with the large sample, z-based confidence interval calculated in Exercise 7.6 (page 262).

7.24 THE BANK CUSTOMER WAITING TIME CASE WaitTime

Using the MINITAB output in Figure 7.15, find a t-based 95 percent confidence interval for the mean of all possible bank customer waiting times. Compare this interval with the large sample, z-based confidence interval calculated in Exercise 7.7 (page 262). Also identify a 95 percent confidence interval for the population median.

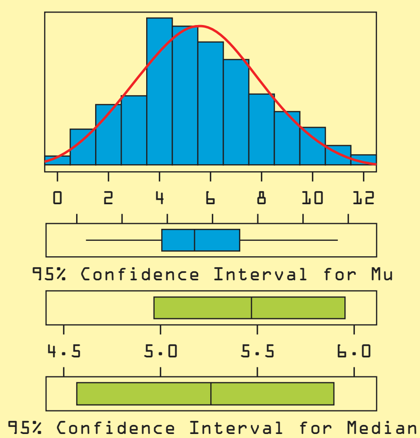
FIGURE 7.13 Excel Output for Exercise 7.22

STATISTICS	
Mean	811
Standard Error	8.786353
Median	814
Mode	#N/A
Standard Deviation	19.64688
Sample Variance	386
Kurtosis	−0.12472
Skewness	−0.23636
Range	52
Minimum	784
Maximum	836
Sum	4055
Count	5
Confidence Level(95.0%)	24.39488

FIGURE 7.14 Excel Output for Exercise 7.23

STATISTICS	
Mean	50.575
Standard Error	0.2599
Median	50.65
Mode	50.9
Standard Deviation	1.643753
Sample Variance	2.701923
Kurtosis	−0.2151
Skewness	−0.05493
Range	7.2
Minimum	46.8
Maximum	54
Sum	2023
Count	40
Confidence Level(95.0%)	0.525697

FIGURE 7.15 MINITAB Output for Exercise 7.24




```
              Variable: WaitTime
      Anderson-Darling Normality Test
        A-Squared:                0.244
        P-Value:                  0.759

        Mean                    5.46000
        StDev                   2.47546
        Variance                6.12788
        Skewness               0.250415
        Kurtosis              -4.0E-01
        N                           100

        Minimum                  0.4000
        1st Quartile             3.8000
        Median                   5.2500
        3rd Quartile             7.2000
        Maximum                 11.6000
     95% Confidence Interval for Mu
            4.9688              5.9512
    95% Confidence Interval for Sigma
           2.1735              2.8757
   95% Confidence Interval for Median
           4.5742              5.8773
```

7.3 Sample Size Determination

In Example 7.1 we used a sample of 49 mileages to construct a 95 percent confidence interval for the midsize model's mean mileage μ. The size of this sample was not arbitrary—it was planned. To understand this, suppose that before the automaker selected the random sample of 49 mileages, it randomly selected the small sample of five mileages that is shown as the leftmost sample in Table 7.1. This sample consists of the mileages

30.8 31.9 30.3 32.1 31.4

and has mean $\bar{x} = 31.3$ and standard deviation $s = .7517$. Assuming that the population of all mileages is mound-shaped, we can calculate a confidence interval for the population mean mileage μ by using the t distribution with $n - 1 = 5 - 1 = 4$ degrees of freedom. It follows that a 95 percent confidence interval for μ is

$$\left[\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}\right] = \left[31.3 \pm 2.776\frac{.7517}{\sqrt{5}}\right]$$
$$= [31.3 \pm .9333]$$
$$= [30.4, 32.2]$$

Although the sample mean $\bar{x} = 31.3$ is at least 31, the lower limit of the 95 percent confidence interval for μ is less than 31. Therefore, the midsize model's EPA mileage estimate would be 30 mpg, and the automaker would not receive its tax credit. One reason that the lower limit of this 95 percent interval is less than 31 is that the sample size of 5 is not large enough to make the *half-length* of the interval

$$t_{.025} \frac{s}{\sqrt{n}} = 2.776 \frac{.7515}{\sqrt{5}} = .9333$$

small enough. If this half-length were .3 or less, the lower limit of the interval would be 31 or more.

We will soon see how to find the size of the sample that will be needed to make the half-length of a confidence interval for μ as small as we wish. When we do this, the needed sample size will probably be large (that is, at least 30). If this is so, we will estimate μ by using the $100(1 - \alpha)$ percent confidence interval based on the normal distribution

$$\left[\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right]$$

Of course, assuming that we do not know σ, we will estimate σ by the large sample's standard deviation s. From the standpoint of developing a formula for the needed sample size, however, we will initially assume that we know σ. The half-length $z_{\alpha/2}(\sigma/\sqrt{n})$ of the confidence interval is also called the **error bound** of the interval. This is because we are $100(1 - \alpha)$ percent confident that $\bar{x}$ is within $z_{\alpha/2}(\sigma/\sqrt{n})$ units of μ. Suppose that in the mileage situation we feel that the mean $\bar{x}$ of the large sample that we will take will be at least 31.3 mpg (the mean of the small sample of five mileages we have already taken). If this is so, then the lower limit of the $100(1 - \alpha)$ percent confidence interval for μ will be at least 31 if the error bound is .3 or less. To find the sample size n that makes the error bound equal to .3, we set $z_{\alpha/2}(\sigma/\sqrt{n})$ equal to .3 and solve for n.

Before solving for n, we must realize that in other problems the *desired* error bound will not equal .3. In general, we will let B denote the desired error bound. Setting the error bound equal to B, we obtain

$$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = B$$

Multiplying both sides of this equation by $\sqrt{n}$ and dividing both sides by B, we obtain

$$\sqrt{n} = \frac{z_{\alpha/2}\sigma}{B}$$

Squaring both sides of this result gives us the formula for n.

Determining the Sample Size for a Confidence Interval for μ

A sample of size

$$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2$$

makes the error bound in a $100(1 - \alpha)$ percent confidence interval for μ equal to B. That is, this sample size makes us $100(1 - \alpha)$ percent confident that $\bar{x}$ is within B units of μ.

If we consider the formula for the sample size n, it intuitively follows that the value B is the farthest that the user is willing to allow $\bar{x}$ to be from μ at a given level of confidence, and the normal point $z_{\alpha/2}$ follows directly from the given level of confidence. Furthermore, because the population standard deviation σ is in the numerator of the formula for n, it follows that the more variable that the individual population measurements are, the larger is the sample size needed to estimate μ with a specified accuracy.

In order to use this formula for n, we must either know σ (which is unlikely) or we must compute an estimate of σ. Often we estimate σ by using a **preliminary sample.** Generally speaking, the following is accepted practice:

1. If the preliminary sample is large (at least 30), we modify the formula for n by simply replacing σ by the standard deviation of the preliminary sample.
2. If the preliminary sample is small (less than 30), we modify the formula for n by replacing σ by the standard deviation of the preliminary sample and by replacing $z_{\alpha/2}$ by $t_{\alpha/2}$. Here the number of degrees of freedom for the $t_{\alpha/2}$ point is the size of the preliminary sample minus 1. Intuitively, using $t_{\alpha/2}$ (which is larger than $z_{\alpha/2}$) compensates for the fact that, since the preliminary sample is small, the standard deviation of the preliminary sample might not be an extremely accurate estimate of σ.

EXAMPLE 7.5 The Car Mileage Case

Suppose that in the car mileage situation we wish to find the sample size that is needed to make the error bound in a 95 percent confidence interval for μ equal to .3. Because we do not know σ, we regard the previously discussed sample of five mileages as a preliminary sample. Therefore, we replace σ by the standard deviation of the preliminary sample (recall that $s = .7517$ for this sample). The preliminary sample is small, so we also replace $z_{\alpha/2} = z_{.025} = 1.96$ by $t_{.025} = 2.776$, which is based on $n - 1 = 4$ degrees of freedom. We find that the appropriate sample size is

$$n = \left(\frac{2.776\sigma}{B}\right)^2 = \left(\frac{2.776(.7517)}{.3}\right)^2 = 48.38$$

Rounding up, we employ a sample of size 49. Here we have rounded up in order to guarantee that the error bound in our 95 percent confidence interval is no more than the desired .3 mpg.

When we make the error bound in our 95 percent confidence interval for μ equal to .3, we can say we are 95 percent confident that the sample mean $\bar{x}$ is within .3 of μ. To understand this, suppose the true value of μ is 31.5. Recalling that the mean of the sample of 49 mileages is $\bar{x} = 31.5531$, we see that this sample mean is within .3 of μ (in fact, it is $31.5531 - 31.5 = .0531$ mpg from $\mu = 31.5$). Other samples of 49 mileages would give different sample means that would be different distances from μ. When we say that our sample of 49 mileages makes us 95 percent confident that $\bar{x}$ is within .3 of μ, we mean that **95 percent of all possible sample means based on 49 mileages are within .3 of μ** and 5 percent of such sample means are not. Therefore, when we randomly select one sample of size 49 and compute its sample mean $\bar{x} = 31.5531$, we can be 95 percent confident that this sample mean is within .3 of μ.

In general, the purpose behind replacing $z_{\alpha/2}$ by $t_{\alpha/2}$ when we are using a small preliminary sample to obtain an estimate of σ is to be **conservative,** so that we compute a sample size that is **at least as large as needed.** Because of this, as we illustrate in the next example, we often obtain an error bound that is even smaller than we have requested.

EXAMPLE 7.6 The Car Mileage Case

To see that the sample of 49 mileages has actually produced a 95 percent confidence interval with an error bound that is as small as we requested, recall that the 49 mileages have mean $\bar{x} = 31.5531$ and standard deviation $s = .7992$. Therefore, as shown in Example 7.1, the 95 percent confidence interval for μ is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\right] = \left[31.5531 \pm 1.96\frac{.7992}{\sqrt{49}}\right]$$
$$= [31.5531 \pm .2238]$$
$$= [31.3, 31.8]$$

We see that the error bound in this interval is .2238. This error bound is less than .3 and is, therefore, even smaller than we asked for. Furthermore, as the automaker had hoped, the sample mean $\bar{x} = 31.5531$ of the sample of 49 mileages turned out to be at least 31.3. Therefore, since the error bound is less than .3, the lower limit of the 95 percent confidence interval is higher than 31 mpg (to be specific, the lower bound is 31.3 mpg). This implies that the midsize model's EPA mileage estimate is 31 mpg. Because of this, the automaker will receive its tax credit. This is the practical implication of finding the appropriate sample size.

Finally, sometimes we do not have a preliminary sample that can be used to estimate σ. In this case we have two alternatives. First, we might estimate σ by using our knowledge about a similar population or process. For instance, the automaker might believe that the standard deviation of the mileages for this year's midsize model is about the same as the standard deviation of the mileages for last year's model. Thus, it might be reasonable to use the best available estimate of σ for last year's model as a preliminary estimate of σ for this year's model. Second, it can be shown that, if we can make a reasonable guess of the range of the population being studied, then a conservatively large estimate of σ is this estimated range divided by 4. For example, if the automaker's design engineers feel that almost all of its midsize cars should get mileages within a range of 5 mpg, then a conservatively large estimate of σ is $5/4 = 1.25$ mpg. Looking at the sample size formula

$$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2$$

we can clearly see that a conservatively large estimate of σ will give us a conservatively large sample size.

Exercises for Section 7.3

CONCEPTS

7.33

7.25 Explain why we call the half-length of a confidence interval its *error bound.* What error are we talking about in the context of an interval for μ?

7.26 Explain exactly what we mean when we say that a sample of size n makes us 99 percent confident that $\bar{x}$ is within B units of μ.

7.27 Why do we usually need to take a preliminary sample when determining the size of the sample needed to make the error bound of a confidence interval equal to B?

METHODS AND APPLICATIONS

7.28 Consider a population having a standard deviation equal to 10. We wish to estimate the mean of this population.

- **a** How large a random sample is needed to construct a 95.44 percent confidence interval for the mean of this population with an error bound equal to 1?
- **b** Suppose that we now take a random sample of the size we have determined in part *a*. If we obtain a sample mean equal to 295, calculate the 95.44 percent confidence interval for the population mean. What is the interval's error bound?

7.29 Referring to Exercise 7.11*a* (page 263), regard the sample of 100 public owner controlled companies for which $s = 32.83$ as a preliminary sample. How large a random sample of public owner-controlled companies is needed to make us

- **a** 95 percent confident that $\bar{x}$, the sample mean audit delay, is within four days of μ, the true mean audit delay?
- **b** 99 percent confident that $\bar{x}$ is within four days of μ?

7.30 Referring to Exercise 7.12*b* (page 264), regard the sample of 500 late replacement buyers for which $s = .66$ as a preliminary sample. How large a sample of late replacement buyers is needed to make us

- **a** 99 percent confident that $\bar{x}$, the sample mean number of dealers visited, is within .04 of μ, the true mean number of dealers visited?
- **b** 99.73 percent confident that $\bar{x}$ is within .05 of μ?

7.31 Referring to Exercise 7.22 (page 271), regard the sample of five trial runs for which $s = 19.64688$ as a preliminary sample. Determine the number of trial runs of the chemical process needed to make us

- **a** 95 percent confident that $\bar{x}$, the sample mean hourly yield, is within eight pounds of the true mean hourly yield μ when catalyst XA-100 is used.
- **b** 99 percent confident that $\bar{x}$ is within five pounds of μ. ChemYield

7.32 Referring to Exercise 7.21 (page 271), regard the sample of 12 stock funds for which $s = .31$ as a preliminary sample. How large a sample of stock funds is needed to make us 95 percent confident that $\bar{x}$, the sample mean annual expense, is within .15 of μ, the true mean annual expense for stock funds?

7.33 Referring to Exercise 7.20 (page 271), regard the sample of 20 stabilization times for which $s = 1.57$ as a preliminary sample. Determine the sample size needed to make us 95 percent confident that $\bar{x}$, the sample mean time required to stabilize the emergency situation, is within .5 seconds of μ, the true mean time required to stabilize the emergency situation using the new display panel.

7.4 Confidence Intervals for a Population Proportion

In Chapter 6, the soft cheese spread producer decided to replace its current spout with the new spout if p, the true proportion of all current purchasers who would stop buying the cheese spread if the new spout were used, is less than .10. Suppose that when 1,000 current purchasers are randomly selected and are asked to try the new spout, 63 say they would stop buying the spread if the new spout were used. The point estimate of the population proportion p is the sample proportion $\hat{p} = 63/1{,}000 = .063$. This sample proportion says we estimate that 6.3 percent of all current purchasers would stop buying the cheese spread if the new spout were used. Since $\hat{p}$ equals .063, we have some evidence that p is less than .10.

In order to see if there is strong evidence that p is less than .10, we can calculate a confidence interval for p. As explained in Chapter 6, if the sample size n is large, then the sampling distribution of the sample proportion $\hat{p}$ is approximately a normal distribution with mean $\mu_{\hat{p}} = p$ and standard deviation $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$. By using the same logic we used in developing confidence intervals for μ, it follows that a $100(1-\alpha)$ percent confidence interval for p is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right]$$

We cannot calculate this interval because, since we do not know p, we do not know $p(1-p)$. However, it can be shown that an unbiased estimate of $p(1-p)/n$ is $\hat{p}(1-\hat{p})/(n-1)$. It follows that a $100(1-\alpha)$ percent confidence interval for p can be calculated as summarized below.

A Large Sample $100(1-\alpha)$ percent Confidence Interval for a Population Proportion p

If the sample size n is large, **a $100(1-\alpha)$ percent confidence interval for the population proportion p** is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}}\right]$$

Here n should be considered large if both $n\hat{p}$ and $n(1-\hat{p})$ are at least 5.[1]

EXAMPLE 7.7 The Cheese Spread Case

In the cheese spread situation, consider calculating a confidence interval for p, the population proportion of purchasers who would stop buying the cheese spread if the new spout were used. In order to see whether the sample size $n = 1{,}000$ is large enough to enable us to use the confidence interval formula just given, recall that the point estimate of p is $\hat{p} = 63/1{,}000 = .063$. Therefore, because $n\hat{p} = 1{,}000(.063) = 63$ and $n(1-\hat{p}) = 1{,}000(.937) = 937$ are both greater

[1] Some statisticians suggest using the more conservative rule that both $n\hat{p}$ and $n(1-\hat{p})$ must be at least 10.

than 5, we can use the confidence interval formula. For example, a 95 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}}\right] = \left[.063 \pm 1.96\sqrt{\frac{(.063)(.937)}{1{,}000-1}}\right]$$
$$= [.063 \pm .0151]$$
$$= [.0479, .0781]$$

This interval says that we are 95 percent confident that between 4.79 percent and 7.81 percent of all current purchasers would stop buying the cheese spread if the new spout were used. Below we give the MegaStat output of this interval. Note that MegaStat employs a divisor of n (here 1,000) under the radical rather than the $n - 1$ divisor we have used in the hand calculation above. This is a fairly common practice, although the $n - 1$ divisor is theoretically correct.

Confidence interval - proportion

1000 n	95% confidence level	**0.078 upper confidence limit**
1.960 z	0.063 proportion	**0.048 lower confidence limit**

A 99 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}}\right] = \left[.063 \pm 2.575\sqrt{\frac{(.063)(.937)}{1{,}000-1}}\right]$$
$$= [.063 \pm .0198]$$
$$= [.0432, .0828]$$

The upper limits of both the 95 percent and 99 percent intervals are less than .10. Therefore, we have very strong evidence that the true proportion p of all current purchasers who would stop buying the cheese spread is less than .10. Based on this result, it seems reasonable to use the new spout.

In the cheese spread example, a sample of 1,000 purchasers gives us a 95 percent confidence interval for p—[.063 ± .0151]—with a reasonably small error bound of .0151. Generally speaking, quite a large sample is needed in order to make the error bound of a confidence interval for p reasonably small. The next two examples demonstrate that a sample size of 200, which most people would consider quite large, does not necessarily give a 95 percent confidence interval for p with a small error bound.

EXAMPLE 7.8

Antibiotics occasionally cause nausea as a side effect. Scientists working for a major drug company have developed a new antibiotic called Phe-Mycin. The company wishes to estimate p, the proportion of all patients who would experience nausea as a side effect when being treated with Phe-Mycin. Suppose that a sample of 200 patients is randomly selected. When these patients are treated with Phe-Mycin, 35 patients experience nausea. The point estimate of the population proportion p is the sample proportion $\hat{p} = 35/200 = .175$. This sample proportion says that we estimate that 17.5 percent of all patients would experience nausea as a side effect of taking Phe-Mycin. Furthermore, because $n\hat{p} = 200(.175) = 35$ and $n(1 - \hat{p}) = 200(.825) = 165$ are both at least 5, we can use the previously given formula to calculate a confidence interval for p. Doing this, we find that a 95 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}}\right] = \left[.175 \pm 1.96\sqrt{\frac{(.175)(.825)}{200-1}}\right]$$
$$= [.175 \pm .053]$$
$$= [.122, .228]$$

This interval says we are 95 percent confident that between 12.2 percent and 22.8 percent of all patients would experience nausea as a side effect of taking Phe-Mycin. Notice that the error bound (.053) of this interval is rather large. Therefore, this interval is fairly long, and it does not provide a very precise estimate of p.

EXAMPLE 7.9 The Marketing Ethics Case

Recall from Example 2.16 (page 86) that we wish to estimate the proportion, p, of all marketing researchers who disapprove of the actions taken in the ultraviolet ink scenario. Also recall that since 117 of the 205 surveyed marketing researchers said that they disapproved, the point estimate of p is the sample proportion $\hat{p} = 117/205 = .5707$. Because $n\hat{p} = 205(.5707) = 117$ and $n(1 - \hat{p}) = 205(.4293) = 88$ are both at least 5, it follows that a 95 percent confidence interval for p is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}}\right] = \left[.5707 \pm 1.96\sqrt{\frac{(.5707)(.4293)}{205-1}}\right]$$
$$= [.5707 \pm .0679]$$
$$= [.5028, .6386]$$

This interval says we are 95 percent confident that between 50.28 percent and 63.86 percent of all marketing researchers disapprove of the actions taken in the ultraviolet ink scenario. Notice that since the error bound (.0679) of this interval is rather large, this interval does not provide a very precise estimate of p. Below we show the MINITAB output of this interval.

```
CI for One Proportion
X      N     Sample p      95.0% CI
117    205   0.5707       (0.5028, 0.6386)
```

In order to find the size of the sample needed to estimate a population proportion, we consider the theoretically correct interval

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}\right]$$

To obtain the sample size needed to make the error bound in this interval equal to B, we set

$$z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} = B$$

and solve for n. When we do this, we get the following result:

Determining the Sample Size for a Confidence Interval for p

A sample of size

$$n = p(1-p)\left(\frac{z_{\alpha/2}}{B}\right)^2$$

makes the error bound in a $100(1-\alpha)$ percent confidence interval for p equal to B. That is, this sample size makes us $100(1-\alpha)$ percent confident that $\hat{p}$ is within B units of p.

Looking at this formula, we see that, the larger $p(1 - p)$ is, the larger n will be. To make sure n is large enough, consider Figure 7.16, which is a graph of $p(1 - p)$ versus p. This figure shows that $p(1 - p)$ equals .25 when p equals .5. Furthermore, $p(1 - p)$ is never larger than .25. Therefore, if the true value of p could be near .5, we should set $p(1 - p)$ equal to .25. This will ensure that n is as large as needed to make the error bound as small as desired (because n appears in the denominator of the formula for the error bound). For example, suppose we wish to estimate the proportion p of all registered voters who currently favor a particular candidate for president of the United States. If this candidate is the nominee of a major political party, or if the candidate enjoys broad popularity for some other reason, then p could be near .5. Furthermore, suppose we wish to make the error bound in a 95 percent confidence interval for p equal to .02. If the sample

FIGURE 7.16 The Graph of $p(1 - p)$ versus p

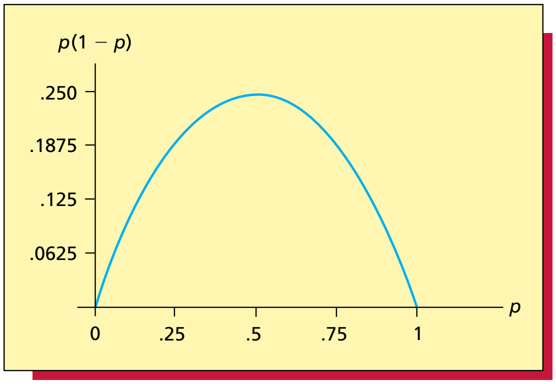


FIGURE 7.17 MegaStat Output of a Sample Size Calculation

Sample size - proportion
0.02 E, error tolerance
0.5 estimated population proportion
95% confidence level
1.960 z
2400.905 sample size
2401 rounded up

to be taken is random, it should consist of

$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{B}\right)^2 = .25\left(\frac{1.96}{.02}\right)^2 = 2{,}401$$

registered voters. The MegaStat output of the results of this calculation is shown in Figure 7.17. In reality, a list of all registered voters in the United States is not available to polling organizations. Therefore, it is not feasible to take a (technically correct) random sample of registered voters. For this reason, polling organizations actually employ other (more complicated) kinds of samples. We explain some of the basic ideas behind these more complex samples in optional Section 7.6. For now, we consider the samples taken by polling organizations to be approximately random. Suppose, then, that when the sample of voters is actually taken, the proportion $\hat{p}$ of sampled voters who favor the candidate turns out to be greater than .52. It follows, because the sample is large enough to make the error bound in a 95 percent confidence interval for p equal to .02, that the lower limit of such an interval is greater than .50. This says we have strong evidence that a majority of all registered voters favor the candidate. For instance, if the sample proportion $\hat{p}$ equals .53, we are 95 percent confident that the proportion of all registered voters who favor the candidate is between .51 and .55.

Major polling organizations conduct public opinion polls concerning many kinds of issues. Whereas making the error bound in a 95 percent confidence interval for p equal to .02 requires a sample size of 2,401, making the error bound in such an interval equal to .03 requires a sample size of only

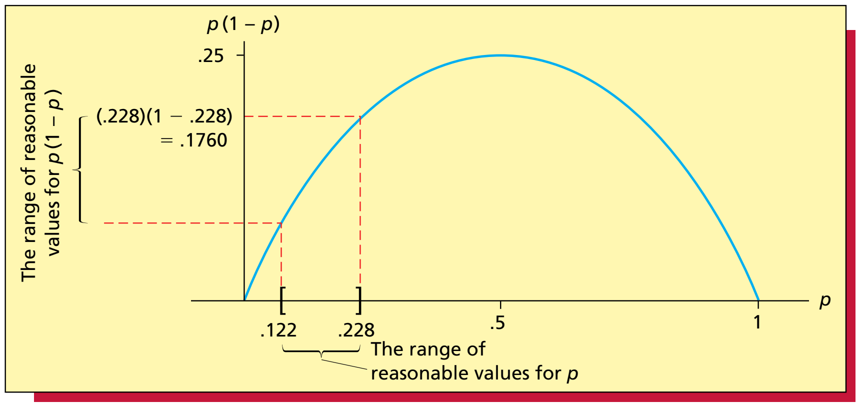
$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{B}\right)^2 = .25\left(\frac{1.96}{.03}\right)^2 = 1{,}067.1$$

or 1,068 (rounding up). Of course, these calculations assume that the proportion p being estimated could be near .5. However, for any value of p, increasing the error bound from .02 to .03 substantially decreases the needed sample size and thus saves considerable time and money. For this reason, although the most accurate public opinion polls use an error bound of .02, the vast majority of public opinion polls use an error bound of .03 or larger.

When the news media report the results of a public opinion poll, they refer to the error bound in a 95 percent confidence interval for p as the poll's *margin of error.* **The margin of error is usually expressed in percentage points.** For instance, if the error bound is .03, the media would say the poll's margin of error is 3 percentage points. The media seldom report the level of confidence, but almost all polling results are based on 95 percent confidence. Sometimes the media make a vague reference to the level of confidence. For instance, if the margin of error is 3 percentage points, the media might say that "the sample result will be within 3 percentage points of the population value in 19 out of 20 samples." Here the "19 out of 20 samples" is a reference to the level of confidence, which is $100(19/20) = 100(.95) = 95$ percent.

As an example, suppose a news report says a recent poll finds that 34 percent of the public favors military intervention in an international crisis, and suppose the poll's margin of error is

FIGURE 7.18 **The Largest Reasonable Value for $p(1 - p)$ in the Antibiotic Example Is $(.228)(1 - .228) = .1760$**



reported to be 3 percentage points. This means the sample taken is large enough to make us 95 percent confident that the sample proportion $\hat{p} = .34$ is within .03 (that is, 3 percentage points) of the true proportion p of the entire public that favors military intervention. That is, we are 95 percent confident that p is between .31 and .37.

If the population proportion we are estimating is substantially different from .5, setting $p(1 - p)$ equal to .25 will give a sample size that is much larger than is needed. In this case, we should use our intuition or previous sample information—along with Figure 7.16—to determine the largest reasonable value for $p(1 - p)$. We then use this value of $p(1 - p)$ to calculate the needed sample size, as is illustrated in the following example.

EXAMPLE 7.10

Again consider estimating the proportion of all patients who would experience nausea as a side effect of taking the new antibiotic Phe-Mycin. Suppose the drug company wishes to find the size of the random sample that is needed in order to be 95 percent confident that $\hat{p}$ is within .02 of p (that is, in order to obtain a 2 percent margin of error with 95 percent confidence). In Example 7.8 we employed a sample of 200 patients to compute a 95 percent confidence interval for p. This interval, which is [.122, .228], makes us very confident that p is between .122 and .228. Figure 7.18 shows that for any range of reasonable values of p that does not contain .5—such as [.122, .228]—the quantity $p(1 - p)$ is maximized by the reasonable value of p that is closest to .5. In the Phe-Mycin situation, .228 is the reasonable value of p that is closest to .5, and it follows that the largest reasonable value of $p(1 - p)$ is $.228(1 - .228) = .1760$. Therefore, the drug company should take a random sample of

$$n = p(1 - p)\left(\frac{z_{\alpha/2}}{B}\right)^2 = .1760\left(\frac{1.96}{.02}\right)^2 = 1{,}691 \text{ (rounded up)}$$

patients.

Finally, as a last example of choosing p for sample size calculations, suppose that experience indicates that a population proportion p is at least .75. Then, .75 is the reasonable value of p that is closest to .5, and we would use the largest reasonable value of $p(1 - p)$, which is $.75(1 - .75) = .1875$.

Exercises for Section 7.4

www.mhhe.com/bowerman3e

7.35, 7.37, 7.39, 7.41, 7.48

CONCEPTS

7.34 **a** What does a population proportion tell us about the population?

b Explain the difference between p and $\hat{p}$.

c What is meant when a public opinion poll's *margin of error* is 3 percent?

7.35 Suppose we are using the sample size formula in the box on page 278 to find the sample size needed to make the error bound in a confidence interval for p equal to B. In each of the following situations, explain what value of p would be used in the formula for finding n:

a We have no idea what value p is—it could be any value between 0 and 1.
b Past experience tells us that p is no more than .3.
c Past experience tells us that p is at least .8.

METHODS AND APPLICATIONS

7.36 In each of the following cases, determine whether the sample size n is large enough to use the large sample formula presented in the box on page 276 to compute a confidence interval for p.

a $\hat{p} = .1, \quad n = 30$ **d** $\hat{p} = .8, \quad n = 400$
b $\hat{p} = .1, \quad n = 100$ **e** $\hat{p} = .9, \quad n = 30$
c $\hat{p} = .5, \quad n = 50$ **f** $\hat{p} = .99, \quad n = 200$

7.37 In each of the following cases, compute 95 percent, 98 percent, and 99 percent confidence intervals for the population proportion p.

a $\hat{p} = .4$ and $n = 100$ **c** $\hat{p} = .9$ and $n = 100$
b $\hat{p} = .1$ and $n = 300$ **d** $\hat{p} = .6$ and $n = 50$

7.38 The February 7, 1999, issue of *The Cincinnati Enquirer* reported the results of a *USA Today*/CNN/Gallup poll concerning surplus spending priorities. The poll was conducted January 22–24, 1999, and asked 1,031 randomly selected adult Americans what their first priority was for spending the \$2.6 trillion in federal budget surpluses projected over the next decade. The two highest priorities were spending on Social Security, which was favored by 23 percent of the polled Americans, and spending on education, which was favored by 21 percent. Find a 95 percent confidence interval for the proportion of all adult Americans who favor spending the surplus on Social Security, and find a 95 percent confidence interval for the proportion of all adult Americans who favor spending the surplus on education.

7.39 THE MARKETING ETHICS CASE: CONFLICT OF INTEREST

Recall that a conflict of interest scenario was presented to a sample of 205 marketing researchers and that 111 of these researchers disapproved of the actions taken in the scenario.

a Assuming that the sample of 205 marketing researchers was randomly selected, use this sample information to show that the 95 percent confidence interval for the proportion of all marketing researchers who disapprove of the actions taken in the conflict of interest scenario is as given in the MINITAB output below. Interpret this interval.

```
CI for One Proportion
X      N     Sample p     95.0% CI
111    205     0.5414     (0.4732, 0.6096)
```

b On the basis of this interval, is there convincing evidence that a majority of all marketing researchers disapprove of the actions taken in the conflict of interest scenario? Explain.

7.40 In a news story distributed by the *Washington Post,* Lew Sichelman reports that a substantial fraction of mortgage loans that go into default within the first year of the mortgage were approved on the basis of falsified applications. For instance, loan applicants often exaggerate their income or fail to declare debts. Suppose that a random sample of 1,000 mortgage loans that were defaulted within the first year reveals that 410 of these loans were approved on the basis of falsified applications.

a Find a point estimate of and a 95 percent confidence interval for p, the proportion of all first-year defaults that are approved on the basis of falsified applications.
b Based on your interval, what is a reasonable estimate of the minimum percentage of first-year defaults that are approved on the basis of falsified applications?

7.41 On January 7, 2000, the Gallup Organization released the results of a poll comparing the lifestyles of today with yesteryear. The survey results were based on telephone interviews with a randomly selected national sample of 1,031 adults,18 years and older, conducted December 20–21, 1999.[2]

a The Gallup poll found that 42 percent of the respondents said that they spend less than three hours watching TV on an average weekday. Based on this finding, calculate a 99 percent confidence interval for the proportion of U.S. adults who say that they spend less than three hours watching TV on an average weekday. Based on this interval, is it reasonable to conclude that more than 40 percent of U.S. adults say they spend less than three hours watching TV on an average weekday?

[2] **Source:** World Wide Web, http://www.gallup.com/poll/releases/, The Gallup Organization, January 7, 2000.

b The Gallup poll found that 60 percent of the respondents said they took part in some form of daily activity (outside of work, including housework) to keep physically fit. Based on this finding, find a 95 percent confidence interval for the proportion of U.S. adults who say they take part in some form of daily activity to keep physically fit. Based on this interval, is it reasonable to conclude that more than 50 percent of U.S. adults say they take part in some form of daily activity to keep physically fit?

c In explaining its survey methods, Gallup states the following: "For results based on this sample, one can say with 95 percent confidence that the maximum error attributable to sampling and other random effects is plus or minus 3 percentage points." Explain how your calculations for part *b* verify that this statement is true.

7.42 In an article in the *Journal of Advertising,* Weinberger and Spotts compare the use of humor in television ads in the United States and the United Kingdom. They found that a substantially greater percentage of U.K. ads use humor.

a Suppose that a random sample of 400 television ads in the United Kingdom reveals that 142 of these ads use humor. Show that the point estimate and 95 percent confidence interval for the proportion of all U.K. television ads that use humor are as given in the MegaStat output below.

Confidence interval - proportion

400 n	95% confidence level	**0.402 upper confidence limit**
1.960 z	0.355 proportion	**0.308 lower confidence limit**

b Suppose a random sample of 500 television ads in the United States reveals that 122 of these ads use humor. Find a point estimate of and a 95 percent confidence interval for the proportion of all U.S. television ads that use humor.

c Do the confidence intervals you computed in parts *a* and *b* suggest that a greater percentage of U.K. ads use humor? Explain. How might an ad agency use this information?

7.43 In an article in *CA Magazine,* Neil Fitzgerald surveyed Scottish business customers concerning their satisfaction with aspects of their banking relationships. Fitzgerald reports that, in 418 telephone interviews conducted by George Street Research, 67 percent of the respondents gave their banks a high rating for overall satisfaction.

a Assuming that the sample is randomly selected, calculate a 99 percent confidence interval for the proportion of Scottish business customers who give their banks a high rating for overall satisfaction.

b Based on this interval, can we be 99 percent confident that more than 60 percent of Scottish business customers give their banks a high rating for overall satisfaction?

7.44 In the March 16, 1998, issue of *Fortune* magazine, the results of a survey of 2,221 MBA students from across the United States conducted by the Stockholm-based academic consulting firm Universum showed that only 20 percent of MBA students expect to stay at their first job five years or more.[3] Assuming that a random sample was employed, find a 95 percent confidence interval for the proportion of all U.S. MBA students who expect to stay at their first job five years or more. Based on this interval, is there strong evidence that fewer than one-fourth of all U.S. MBA students expect to stay?

7.45 In Exercise 2.11 (page 54), we briefly described a series of international quality standards called ISO 9000. In the results of a Quality Systems Update/Deloitte & Touche survey of ISO 9000 registered companies published by CEEM Information Systems, 515 of 620 companies surveyed reported that they are encouraging their suppliers to pursue ISO 9000 registration.[4]

a Using these survey results, compute a 95.44 percent confidence interval for the proportion of all ISO 9000 registered companies that encourage their suppliers to pursue ISO 9000 registration. Assume here that the survey participants have been randomly selected.

b Based on this interval, is there conclusive evidence that more than 75 percent of all ISO 9000 registered companies encourage their suppliers to pursue ISO 9000 registration?

7.46 The manufacturer of the ColorSmart-5000 television set claims 95 percent of its sets last at least five years without needing a single repair. In order to test this claim, a consumer group randomly selects 400 consumers who have owned a ColorSmart-5000 television set for five years. Of these 400 consumers, 316 say their ColorSmart-5000 television sets did not need a repair, whereas 84 say their ColorSmart-5000 television sets did need at least one repair.

a Find a 99 percent confidence interval for the proportion of all ColorSmart-5000 television sets that have lasted at least five years without needing a single repair.

[3]**Source:** Shelly Branch, "MBAs: What Do They Really Want," *Fortune* (March 16, 1998), p. 167.

[4]**Source:** *Is ISO 9000 for You?* (Fairfax, VA: CEEM Information Services).

b Does this confidence interval provide strong evidence that the percentage of ColorSmart-5000 television sets that last at least five years without a single repair is less than the 95 percent claimed by the manufacturer? Explain.

7.47 In the book *Cases in Finance,* Nunnally and Plath present a case in which the estimated percentage of uncollectible accounts varies with the age of the account. Here the age of an unpaid account is the number of days elapsed since the invoice date.

Suppose an accountant believes the percentage of accounts that will be uncollectible increases as the ages of the accounts increase. To test this theory, the accountant randomly selects 500 accounts with ages between 31 and 60 days from the accounts receivable ledger dated one year ago. The accountant also randomly selects 500 accounts with ages between 61 and 90 days from the accounts receivable ledger dated one year ago.

a If 10 of the 500 accounts with ages between 31 and 60 days were eventually classified as uncollectible, find a point estimate of and a 95 percent confidence interval for the proportion of all accounts with ages between 31 and 60 days that will be uncollectible.

b If 27 of the 500 accounts with ages between 61 and 90 days were eventually classified as uncollectible, find a point estimate of and a 95 percent confidence interval for the proportion of all accounts with ages between 61 and 90 days that will be uncollectible.

c Based on these intervals, is there strong evidence that the percentage of accounts aged between 61 and 90 days that will be uncollectible is higher than the percentage of accounts aged between 31 and 60 days that will be uncollectible? Explain.

7.48 Consider Exercise 7.41*b* and suppose we wish to find the sample size n needed in order to be 95 percent confident that $\hat{p}$, the sample proportion of respondents who said they took part in some sort of daily activity to keep physically fit, is within .02 of p, the true proportion of all U.S. adults who say that they take part in such activity. In order to find an appropriate value for $p(1 - p)$, note that the 95 percent confidence interval for p that you calculated in Exercise 7.41*b* was [.57, .63]. This indicates that the reasonable value for p that is closest to .5 is .57, and thus the largest reasonable value for $p(1 - p)$ is $.57(1 - .57) = .2451$. Calculate the required sample size n.

7.49 Referring to Exercise 7.46, determine the sample size needed in order to be 99 percent confident that $\hat{p}$, the sample proportion of ColorSmart-5000 television sets that last at least five years without a single repair, is within .03 of p, the true proportion of sets that last at least five years without a single repair.

7.50 Suppose we conduct a poll to estimate the proportion of voters who favor a major presidential candidate. Assuming that 50 percent of the electorate could be in favor of the candidate, determine the sample size needed so that we are 95 percent confident that $\hat{p}$, the sample proportion of voters who favor the candidate, is within .01 of p, the true proportion of all voters who are in favor of the candidate.

*7.5 Confidence Intervals for Parameters of Finite Populations

It is best to use the confidence intervals presented in Sections 7.1 through 7.4 when the sampled population is either infinite or finite and *much larger than* (say, at least 20 times as large as) the sample. Although these previously discussed intervals are sometimes used when a finite population is not much larger than the sample, better methods exist for handling such situations. We present these methods in this section.

As we have explained, we often wish to estimate a population mean. Sometimes we also wish to estimate a *population total.*

A **population total** is the sum of the values of all the population measurements.

For example, companies in financial trouble have sometimes falsified their accounts receivable invoices in order to mislead stockholders. For this reason, independent auditors are often asked to estimate a company's true total sales for a given period. The auditor randomly selects a sample of invoices from the population of all invoices, and then independently determines the actual amount of each sale by contacting the purchasers. The sample results are used to estimate the company's total sales, and this estimate can then be compared with the total sales reported by the company.

In order to estimate **a population total, which we denote as τ (pronounced "tau"),** we note that the population mean μ is the population total divided by the number, N, of population measurements. That is, we have $\mu = \tau/N$, which implies that $\tau = N\mu$. It follows, because a point estimate of the population mean μ is the sample mean $\bar{x}$, that

A **point estimate of a population total τ** is $N\bar{x}$, where N is the size of the population.

EXAMPLE 7.11

A company sells and installs satellite dishes and receivers for both private individuals and commercial establishments (bars, restaurants, and so forth). The company accumulated 2,418 sales invoices during last year. The total of the sales amounts listed on these invoices (that is, the total sales claimed by the company) is $5,127,492.17. In order to estimate the true total sales, τ, for last year, an independent auditor randomly selects 242 of the invoices and determines the actual sales amounts by contacting the purchasers. When the sales amounts are averaged, the mean of the actual sales amounts for the 242 sampled invoices is $\bar{x} = \$1{,}843.93$. This says that a point estimate of the true total sales τ is

$$N\bar{x} = 2{,}418(\$1{,}843.93) = \$4{,}458{,}622.70$$

This point estimate is considerably lower than the claimed total sales of $5,127,492.17. However, we cannot expect the point estimate of τ to exactly equal the true total sales, so we need to calculate a confidence interval for τ before drawing any unwarranted conclusions.

In order to find a confidence interval for the mean and total of a finite population, we consider the sampling distribution of the sample mean $\bar{x}$. It can be shown that, if we randomly select a large sample of n measurements without replacement from a finite population of N measurements, then the sampling distribution of $\bar{x}$ is approximately normal with mean $\mu_{\bar{x}} = \mu$ and standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

It can also be shown that the appropriate point estimate of $\sigma_{\bar{x}}$ is $(s/\sqrt{n})(\sqrt{(N-n)/N})$, where s is the sample standard deviation. This point estimate of $\sigma_{\bar{x}}$ is used in the confidence intervals for μ and τ, which we summarize as follows:

Confidence Intervals for the Population Mean and Population Total for a Finite Population

Suppose we randomly select a sample of n measurements **without replacement from a finite population of N measurements.** Then, if n is large (say, at least 30)

1 **A $100(1 - \alpha)$ percent confidence interval for the population mean μ** is

$$\left[\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N}}\right]$$

2 **A $100(1 - \alpha)$ percent confidence interval for the population total τ** is found by multiplying the lower and upper limits of the $100(1 - \alpha)$ percent confidence interval for μ by N.

The quantity $\sqrt{(N-n)/N}$ in the confidence intervals for μ and τ is called the **finite population correction.** If the population size N is much larger than (say, at least 20 times as large as) the sample size n, then the finite population correction is approximately equal to 1. For example, if we randomly select (without replacement) a sample of 1,000 from a population of 1 million, then the finite population correction is $\sqrt{(1{,}000{,}000 - 1{,}000)/1{,}000{,}000} = .9995$. In such a case, many people believe it is not necessary to include the finite population correction in the confidence interval calculations. This is because the correction is not far enough below 1 to meaningfully shorten the confidence intervals for μ and τ. However, **if the population size N**

is not much larger than the sample size n (say, if n is more than 5 percent of N), then the finite population correction is substantially less than 1 and should be included in the confidence interval calculations.

EXAMPLE 7.12

Recall that the satellite dish dealer claims that its total sales τ for last year were \$5,127,492.17. Since the company accumulated 2,418 invoices during last year, the company is claiming that μ, the mean sales amount per invoice, is \$5,127,492.17/2,418 = \$2,120.55. Suppose when the independent auditor randomly selects a sample of $n = 242$ invoices, the mean and standard deviation of the actual sales amounts for these invoices are $\bar{x} = 1{,}843.93$ and $s = 516.42$. Here the sample size $n = 242$ is $(242/2{,}418)100 = 10.008$ percent of the population size $N = 2{,}418$. Because n is more than 5 percent of N, we should include the finite population correction in our confidence interval calculations. It follows that a 95 percent confidence interval for the mean sales amount μ per invoice is

$$\left[\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N}}\right] = \left[1{,}843.93 \pm 1.96\frac{516.42}{\sqrt{242}}\sqrt{\frac{2{,}418-242}{2{,}418}}\right]$$
$$= [1{,}843.93 \pm 61.723812]$$
$$= [1{,}782.21,\ 1{,}905.65]$$

The upper limit of this interval is less than the mean amount of \$2,120.55 claimed by the company, so we have strong evidence that the company is overstating its mean sales per invoice for last year. A 95 percent confidence interval for the total sales τ last year is found by multiplying the lower and upper limits of the 95 percent confidence interval for μ by $N = 2{,}418$. Therefore, this interval is [1,782.21(2,418), 1,905.65(2,418)], or [4,309,383.8, 4,607,861.7]. Because the upper limit of this interval is more than \$500,000 below the total sales amount of \$5,127,492.17 claimed by the company, we have strong evidence that the satellite dealer is substantially overstating its total sales for last year.

We sometimes estimate the total number, τ, of population units that fall into a particular category. For instance, the auditor of Examples 7.11 and 7.12 might wish to estimate the total number of the 2,418 invoices having incorrect sales amounts. Here the proportion, p, of the population units that fall into a particular category is the total number, τ, of population units that fall into the category divided by the number, N, of population units. That is, $p = \tau/N$, which implies that $\tau = Np$. Therefore, since a point estimate of the population proportion p is the sample proportion $\hat{p}$, a point estimate of the population total τ is $N\hat{p}$. For example, suppose that 34 of the 242 sampled invoices have incorrect sales amounts. Because the sample proportion is $\hat{p} = 34/242 = .1405$, a point estimate of the total number of the 2,418 invoices that have incorrect sales amounts is

$$N\hat{p} = 2{,}418(.1405) = 339.729$$

We now summarize how to find confidence intervals for p and τ.

Confidence Intervals for the Proportion of and Total Number of Units in a Category When Sampling a Finite Population

Suppose that we randomly select a sample of n units **without replacement from a finite population of N units.** Then, if n is large

1 **A $100(1-\alpha)$ percent confidence interval for the population proportion p** is

$$\left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)}\right]$$

2 **A $100(1-\alpha)$ percent confidence interval for the population total τ** is found by multiplying the lower and upper limits of the $100(1-\alpha)$ percent confidence interval for p by N.

EXAMPLE 7.13

Recall that in Examples 7.11 and 7.12 we found that 34 of the 242 sampled invoices have incorrect sales amounts. Since $\hat{p} = 34/242 = .1405$, a 95 percent confidence interval for the proportion of the 2,418 invoices that have incorrect sales amounts is

$$\left[\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)}\right] = \left[.1405 \pm 1.96\sqrt{\frac{(.1405)(.8595)}{241}\left(\frac{2{,}418-242}{2{,}418}\right)}\right]$$
$$= [.1405 \pm .0416208]$$
$$= [.0989,\ .1821]$$

This interval says we are 95 percent confident that between 9.89 percent and 18.21 percent of the invoices have incorrect sales amounts. A 95 percent confidence interval for the total number of the 2,418 invoices that have incorrect sales amounts is found by multiplying the lower and upper limits of the 95 percent confidence interval for p by $N = 2{,}418$. Therefore, this interval is [.0989(2,418), .1821(2,418)], or [239.14, 440.32], and we are 95 percent confident that between (roughly) 239 and 440 of the 2,418 invoices have incorrect sales amounts.

Finally, we can determine the sample size that is needed to make the error bound in a confidence interval for μ, p, or τ equal to a desired size B by setting the appropriate error bound formula equal to B and by solving the resulting equation for the sample size n. We will not carry out the details in this book, but the procedure is the same as illustrated in Sections 7.3 and 7.4. Exercise 7.57 gives the reader an opportunity to use the sample size formulas that are obtained.

Exercises for Section 7.5

CONCEPTS

7.51 Define a population total. Give an example of a population total that will interest you in your career when you graduate from college.

7.53

7.52 Explain why the finite population correction $\sqrt{(N-n)/N}$ is unnecessary when the population is at least 20 times as large as the sample. Give an example using numbers.

METHODS AND APPLICATIONS

7.53 A retailer that sells home entertainment systems accumulated 10,451 sales invoices during the previous year. The total of the sales amounts listed on these invoices (that is, the total sales claimed by the company) is \$6,384,675. In order to estimate the true total sales for last year, an independent auditor randomly selects 350 of the invoices and determines the actual sales amounts by contacting the purchasers. The mean and the standard deviation of the 350 sampled sales amounts are $\bar{x} = \$532$ and $s = \$168$.

a Find a 95 percent confidence interval for μ, the true mean sales amount per invoice on the 10,451 invoices.

b Find a point estimate of and a 95 percent confidence interval for τ, the true total sales for the previous year.

c What does this interval say about the company's claim that the true total sales were \$6,384,675? Explain.

7.54 A company's manager is considering simplification of a travel voucher form. In order to assess the costs associated with erroneous travel vouchers, the manager must estimate the total number of such vouchers that were filled out incorrectly in the last month. In a random sample of 100 vouchers drawn without replacement from the 1,323 travel vouchers submitted in the last month, 31 vouchers were filled out incorrectly.

a Find a point estimate of and a 95 percent confidence interval for the true proportion of travel vouchers that were filled out incorrectly in the last month.

b Find a point estimate of and a 95 percent confidence interval for the total number of travel vouchers that were filled out incorrectly in the last month.

c If it costs the company \$10 to correct an erroneous travel voucher, find a reasonable estimate of the minimum cost of correcting all of last month's erroneous travel vouchers. Would it be worthwhile to spend \$5,000 to design a simplified travel voucher that could be used for at least a year?

7.55 A personnel manager is estimating the total number of person-days lost to unexcused absences by hourly workers in the last year. In a random sample of 50 employees drawn without replacement from the 687 hourly workers at the company, records show that the 50 sampled workers had an average of $\bar{x} = 4.3$ days of unexcused absences over the past year with a standard deviation of $s = 1.26$.

a Find a point estimate of and a 95 percent confidence interval for the total number of unexcused absences by hourly workers in the last year.

b Can the personnel manager be 95 percent confident that more than 2,500 person-days were lost to unexcused absences last year? Can the manager be 95 percent confident that more than 3,000 person-days were lost to unexcused absences last year? Explain.

7.56 An auditor randomly samples 32 accounts receivable without replacement from a firm's 600 accounts and checks to verify that all documents for the accounts comply with company procedures. Ten of the 32 accounts are found to have documents not in compliance. Find a point estimate of and a 95 percent confidence interval for the total number of accounts having documents that do not comply with company procedures.

7.57 SAMPLE SIZES WHEN SAMPLING FINITE POPULATIONS

a *Estimating μ and τ*

Consider randomly selecting a sample of n measurements without replacement from a finite population consisting of N measurements and having variance σ^2. Also consider the sample size given by the formula

$$n = \frac{N\sigma^2}{(N - 1)D + \sigma^2}$$

Then, it can be shown that this sample size makes the error bound in (that is, the half-length of) a $100(1 - \alpha)$ percent confidence interval for μ equal to B if we set D equal to $(B/z_{\alpha/2})^2$. It can also be shown that this sample size makes the error bound in a $100(1 - \alpha)$ percent confidence interval for τ equal to B if we set D equal to $[B/(z_{\alpha/2}N)]^2$. Now consider Exercise 7.55. Using $s^2 = (1.26)^2$, or 1.5876, as an estimate of σ^2, determine the sample size that makes the error bound in a 95 percent confidence interval for the *total number* of person-days lost to unexcused absences last year equal to 100 days.

b *Estimating p and τ*

Consider randomly selecting a sample of n units without replacement from a finite population consisting of N units and having a proportion p of these units fall into a particular category. Also, consider the sample size given by the formula

$$n = \frac{Np(1 - p)}{(N - 1)D + p(1 - p)}$$

It can be shown that this sample size makes the error bound in (that is, the half-length of) a $100(1 - \alpha)$ percent confidence interval for p equal to B if we set D equal to $(B/z_{\alpha/2})^2$. It can also be shown that this sample size makes the error bound in a $100(1 - \alpha)$ percent confidence interval for τ equal to B if we set D equal to $[B/(z_{\alpha/2}N)]^2$. Now consider Exercise 7.54. Using $\hat{p} = .31$ as an estimate of p, determine the sample size that makes the error bound in a 95 percent confidence interval for the *proportion* of the 1,323 vouchers that were filled out incorrectly equal to .04.

*7.6 An Introduction to Survey Sampling

Random sampling is not the only kind of sampling. Methods for obtaining a sample are called **sampling designs,** and the sample we take is sometimes called a **sample survey.** In this section we explain three sampling designs that are alternatives to random sampling—**stratified random sampling, cluster sampling,** and **systematic sampling.**

In order to select a **stratified random sample,** we partition (that is, divide) the population into nonoverlapping subpopulations called **strata.** Then a random sample is selected from each stratum. It is wise to stratify when the population consists of two or more subpopulations that differ from each other with respect to the characteristic under study. In other words, the strata should be defined so that the between-strata variability of the characteristic under study is larger than the variability of the characteristic within each stratum.

As an example, suppose that a department store chain proposes to open a new store in a location that would serve customers who live in a geographical region that consists of (1) an industrial city, (2) a suburban community, and (3) a rural area. In order to assess the potential profitability of the proposed store, the chain estimates the mean income and the total income of all homes in the region. In addition, the chain estimates the proportion and the total number of the homes whose occupants would be *likely* to shop at the store. The department store chain feels that the industrial city, the suburban community, and the rural area differ with respect to income and the store's potential desirability. Therefore, it uses these subpopulations as strata and takes a stratified random sample.

In general, stratified random sampling has at least two potential advantages:

1. If the strata can be defined so that the variability of the characteristic under study is larger between the strata than it is within each stratum, stratified sampling will give shorter confidence intervals—for the same sample size—than random sampling will give.
2. Stratifying can make the sample easier to select. Recall that, in order to take a random sample, we must have a frame, or list, of all of the population units. Although a frame might not exist for the overall population, a frame might exist for each stratum. For example, suppose nearly all the homes in the department store's geographical region have telephones. Although there might not be a telephone directory for the overall geographical region, there might be separate telephone directories for the industrial city, the suburb, and the rural area.

We illustrate the formulas used to analyze data from stratified random sampling in Appendix F (Part 1). For a more detailed discussion of stratified random sampling, see Mendenhall, Schaeffer, and Ott (1986).

When samples are taken from very large geographical regions, frames often do not exist. For instance, there is no single list of all registered voters in the United States. There is also no single list of all households in the United States. For this reason, we often use **multistage cluster sampling** to take samples from large geographical regions. To illustrate this procedure, suppose we wish to take a sample of registered voters from all registered voters in the United States. We might proceed as follows:

Stage 1: Randomly select a sample of counties from all of the counties in the United States.

Stage 2: Randomly select a sample of townships from each county selected in Stage 1.

Stage 3: Randomly select a sample of voting precincts from each township selected in Stage 2.

Stage 4: Randomly select a sample of registered voters from each voting precinct selected in Stage 3.

We use the term *cluster sampling* to describe this type of sampling because at each stage we "cluster" the voters into subpopulations. For instance, in Stage 1 we cluster the voters into counties, and in Stage 2 we cluster the voters in each selected county into townships. Also, notice that the random sampling at each stage can be carried out because there are lists of (1) all counties in the United States, (2) all townships in each county, (3) all voting precincts in each township, and (4) all registered voters in each voting precinct.

As another example, consider sampling the households in the United States. We might use Stages 1 and 2 above to select counties and townships within the selected counties. Then, if there is a telephone directory of the households in each township, we can randomly sample households from each selected township by using its telephone directory. Because *most* households today have telephones, and telephone directories are readily available, most national polls are now conducted by telephone.

It is sometimes a good idea to combine stratification with multistage cluster sampling. For example, suppose a national polling organization wants to estimate the proportion of all registered voters who favor a particular presidential candidate. Because the presidential preferences of voters might tend to vary by geographical region, the polling organization might divide the United States into regions (say, Eastern, Midwestern, Southern, and Western regions). The polling organization might then use these regions as strata, and might take a multistage cluster sample from each stratum (region).

Data from multistage cluster sampling are analyzed with formulas that can be quite complicated. We present some simpler versions of these formulas—the ones for one- and two-stage cluster sampling—in Appendix F (Part 2) on the CD-ROM that accompanies this book. This appendix also includes a discussion of an additional survey sampling technique called *ratio estimation.* For a more detailed discussion of cluster sampling and ratio estimation, see Mendenhall, Schaeffer, and Ott (1986).

In order to select a random sample, we must number the units in a frame, or list, of all the population units. Then we use a random number table (or a random number generator on a computer) to make the selections. However, numbering all the population units can be quite time-consuming. Moreover, random sampling is used in the various stages of many complex sampling designs (requiring the numbering of numerous populations). Therefore, it is useful to have an alternative to random sampling. One such alternative is called **systematic sampling.** In order to systematically select a sample of n units without replacement from a frame of N units, we divide N by n and round the result down to the nearest whole number. Calling the rounded result ℓ, we then randomly select one unit from the first ℓ units in the frame—this is the first unit in the systematic sample. The remaining units in the sample are obtained by selecting every ℓth unit following the first (randomly selected) unit. For example, suppose we wish to estimate the average number of times that a population of $n = 14{,}327$ allergists prescribed a particular drug during the last year. A medical society has a directory listing the 14,327 allergists, and we draw a systematic sample of 500 allergists from this frame. Here we compute $14{,}327/500 = 28.654$, which is 28 when rounded down. Therefore, we number the first 28 allergists in the directory from 1 to 28, and we use a random number table to randomly select one of the first 28 allergists. Suppose we select allergist number 19. We interview allergist 19 and every 28th allergist in the frame thereafter, so we choose allergists 19, 47, 75, and so forth until we obtain our sample of 500 allergists. In this scheme, we must number the first 28 allergists, but we do not have to number the rest because we can "count off" every 28th allergist in the directory. Alternatively, we can measure the approximate amount of space in the directory that it takes to list 28 allergists. This measurement can then be used to select every 28th allergist.

If the order of the population units in a frame is random with respect to the characteristic under study, then a systematic sample should be (approximately) a random sample. In this case, we can use the formulas based on random sampling to estimate population parameters. For instance, it would seem reasonable to assume that the alphabetically ordered allergists in a medical directory would be random (that is, have nothing to do with) the number of times the allergists prescribed a particular drug. Similarly, the alphabetically ordered people in a telephone directory would probably be random with respect to many of the people's characteristics that we might wish to study.

Finally, it is important to point out some potential problems with survey samples. First, if we do not have a complete, accurate list of all the population units, there will be some degree of **undercoverage.** For example, although telephone polls today are common, 7 to 8 percent of the people in the United States do not have telephones. In general, undercoverage usually causes low-income people to be underrepresented. If underrepresented groups differ from the rest of the population with respect to the characteristic under study, the survey results will be biased. Second, **nonresponse** can be a serious problem. In some surveys, 25 to 35 percent of the selected individuals cannot be contacted—even when several callbacks are made. In such a case, other participants are often substituted for the people who cannot be contacted. If the substitute participants differ from the originally selected participants with respect to the characteristic under study, the survey will again be biased. Third, when people are asked potentially embarrassing questions, their responses might not be truthful. We then have what we call **response bias.** Fourth, the wording of the questions asked can influence the answers received. Slanted questions often evoke biased responses. For example, consider the following question:

Which of the following best describes your views on gun control?

1 The government should take away our guns, leaving us defenseless against heavily armed criminals.

2 We have the right to keep and bear arms.

This question is biased toward eliciting a response against gun control.

Exercises for Section 7.6

CONCEPTS

7.58 When is it appropriate to use stratified random sampling? What are strata, and how should strata be selected?

7.59 When is cluster sampling used? Why do we describe this type of sampling by using the term *cluster?*

7.60 Explain each of the following terms:
a Undercoverage **b** Nonresponse **c** Response bias

7.61 Explain how to take a systematic sample of 100 companies from the 1,853 companies that are members of an industry trade association.

7.62 Explain how a stratified random sample is selected. Discuss how you might define the strata to survey student opinion on a proposal to charge all students a $100 fee for a new university-run bus system that will provide transportation between off-campus apartments and campus locations.

7.63 Marketing researchers often use city blocks as clusters in cluster sampling. Using this fact, explain how a market researcher might use multistage cluster sampling to select a sample of consumers from all cities having a population of more than 10,000 in a large state having many such cities.

*7.7 A Comparison of Confidence Intervals and Tolerance Intervals

In this section we compare confidence intervals with tolerance intervals. We saw in Chapter 2 (page 68) that a tolerance interval is an interval that is meant to contain a specified percentage (often 68.26 percent, 95.44 percent, or 99.73 percent) of the **individual** population measurements. By contrast, a confidence interval for the population mean μ is an interval that is meant to contain one thing—the population mean μ—and the confidence level associated with the confidence interval expresses how sure we are that this interval contains μ. Often we choose the confidence level to be 95 percent or 99 percent because such a confidence level is usually considered high enough to provide convincing evidence about the true value of μ.

EXAMPLE 7.14 The Car Mileage Case

Recall in the car mileage case that the mean and the standard deviation of the sample of 49 mileages are $\bar{x} = 31.5531$ and $s = .7992$. Also, recall that we have concluded in Example 2.13 (page 68) that the estimated tolerance intervals $[\bar{x} \pm s] = [30.8, 32.4]$, $[\bar{x} \pm 2s] = [30.0, 33.2]$, and $[\bar{x} \pm 3s] = [29.2, 34.0]$ imply that approximately (1) 68.26 percent of all individual cars will obtain mileages between 30.8 mpg and 32.4 mpg; (2) 95.44 percent of all individual cars will obtain mileages between 30.0 mpg and 33.2 mpg; and (3) 99.73 percent of all individual cars will obtain mileages between 29.2 mpg and 34.0 mpg. By contrast, we have seen in Example 7.1 that a 95 percent confidence interval for the mean, μ, of the mileages of all individual cars is $[\bar{x} \pm 1.96(s/\sqrt{49})] = [31.3, 31.8]$. This interval says that we are 95 percent confident that μ is between 31.3 mpg and 31.8 mpg. Figure 7.19 graphically depicts the three estimated tolerance intervals and the 95 percent confidence interval, which are shown below a MINITAB histogram of the 49 mileages. Note that the estimated tolerance intervals, which are meant to contain the *many* mileages that comprise specified percentages of all individual cars, are longer than the 95 percent confidence interval, which is meant to contain the *single* population mean μ.

Exercises for Section 7.7

CONCEPTS

7.68

7.64 What is a tolerance interval meant to contain?

7.65 What is a confidence interval for the population mean meant to contain?

7.66 Intuitively, why is a tolerance interval longer than a confidence interval?

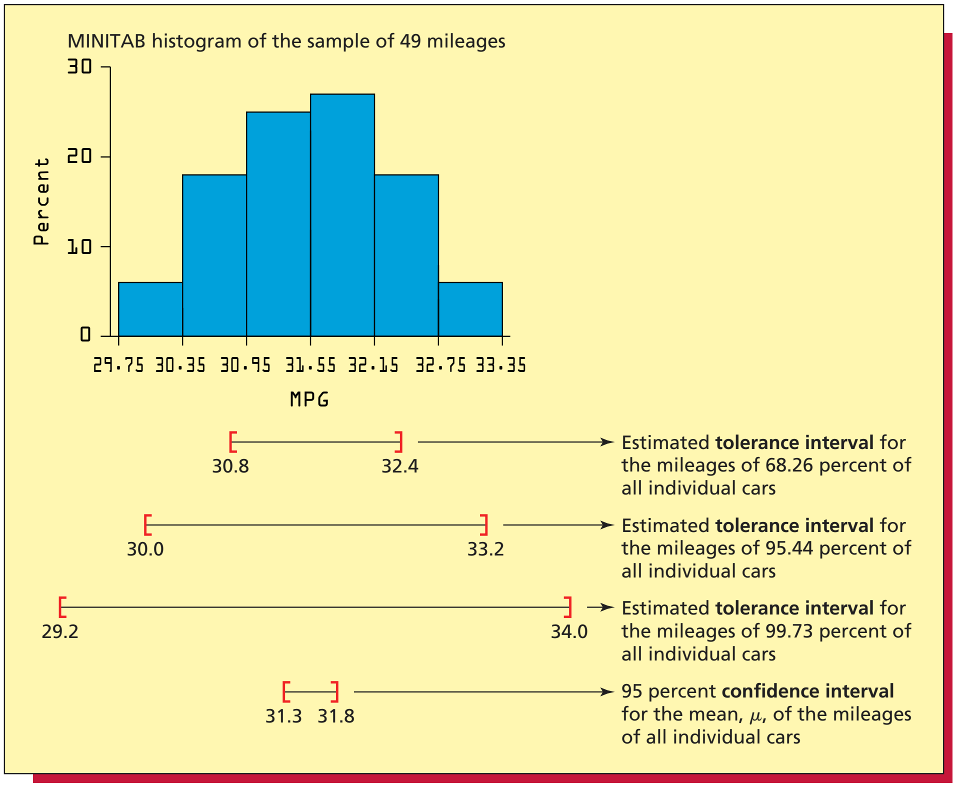
METHODS AND APPLICATIONS

In Exercises 7.67 through 7.69 we give the mean and the standard deviation of a sample that has been randomly selected from a population. For each exercise, find estimated tolerance intervals that contain approximately 68.26 percent, 95.44 percent, and 99.73 percent of the individual population measurements. Also, find a 95 percent confidence interval for the population mean. Interpret the estimated tolerance intervals and the confidence interval in the context of the situation related to the exercise.

7.67 THE TRASH BAG CASE TrashBag

The mean and the standard deviation of the sample of 40 trash bag breaking strengths are $\bar{x} = 50.575$ and $s = 1.6438$.

FIGURE 7.19 A Comparison of Confidence Intervals and Tolerance Intervals



7.68 THE BANK CUSTOMER WAITING TIME CASE WaitTime

The mean and the standard deviation of the sample of 100 bank customer waiting times are $\bar{x} = 5.46$ and $s = 2.475$.

7.69 THE CUSTOMER SATISFACTION RATING CASE CustSat

The mean and the standard deviation of the sample of 65 customer satisfaction ratings are $\bar{x} = 42.95$ and $s = 2.6424$.

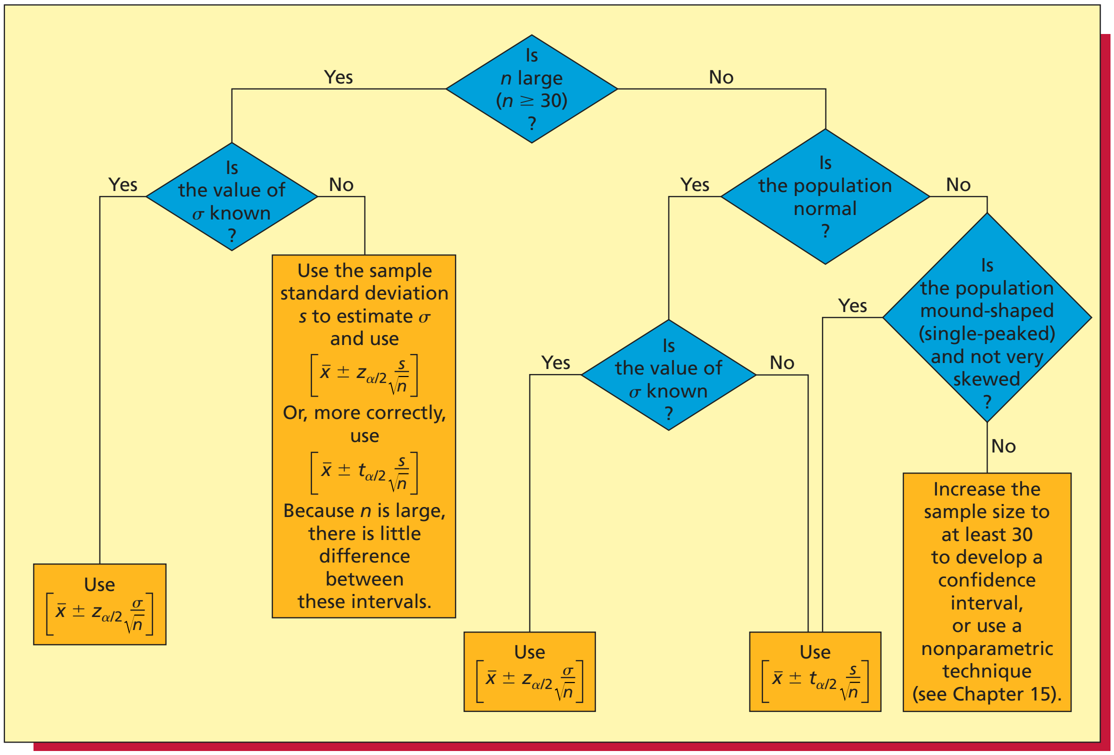
Chapter Summary

In this chapter we discussed **confidence intervals** for population **means, proportions,** and **totals.** We began by assuming that the population is either infinite or much larger than (say, at least 20 times as large as) the sample. First, we studied how to compute a confidence interval for a **population mean.** We saw that when the sample size is at least 30, we can use the **normal distribution** to compute a confidence interval for a population mean. When the sample size is less than 30, if the population is normally distributed (or at least mound-shaped), we use the ***t* distribution** to compute this interval. We also studied how to find the size of the sample needed if we wish to compute a confidence interval for a mean with a prespecified *confidence level* and with a prespecified *error bound.* Figure 7.20 is a flowchart summarizing our discussions concerning how to compute an appropriate confidence interval for a population mean.

Next we saw that we are often interested in estimating the proportion of population units falling into a category of interest. We showed how to compute a large sample confidence interval for a **population proportion,** and we saw how to find the sample size needed to estimate a population proportion with a prespecified *confidence level* and with a prespecified *error bound* or *margin of error.*

We continued by studying how to compute confidence intervals for parameters of **finite populations** that are not much larger than the sample. We saw how to compute confidence intervals for a population mean and total when we are sampling

FIGURE 7.20 Computing an Appropriate Confidence Interval for a Population Mean



without replacement. We also saw how to compute confidence intervals for a population proportion and for the total number of units in a category when sampling a finite population. We concluded this chapter by discussing three alternative sampling designs—**stratified random sampling, cluster sampling,** and **systematic sampling**—and by comparing confidence intervals with tolerance intervals.

Glossary of Terms

cluster sampling (multistage cluster sampling): A sampling design in which we sequentially cluster population units into subpopulations. (page 288)

confidence coefficient: The (before sampling) probability that a confidence interval for a population parameter will contain the population parameter. (page 257)

confidence interval: An interval of numbers computed so that we can be very confident (say, 95 percent confident) that a population parameter is contained in the interval. (page 255)

confidence level: The percentage of time that a confidence interval would contain a population parameter if all possible samples were used to calculate the interval. (page 258)

degrees of freedom (for a *t* curve): A parameter that describes the exact spread of the curve of a *t* distribution. (page 264)

error bound: The half-length of a confidence interval. It gives the maximum distance between the population parameter of interest and its point estimate when we assume the parameter is inside the confidence interval. (page 273)

nonresponse: A situation in which population units selected to participate in a survey do not respond to the survey instrument. (page 289)

population total: The sum of the values of all the population measurements. (page 283)

response bias: A situation in which survey participants do not respond truthfully to the survey questions. (page 289)

standard error of the estimate $\bar{x}$: The point estimate of $\sigma_{\bar{x}}$. (page 269)

strata: The subpopulations in a stratified sampling design. (page 287)

stratified random sampling: A sampling design in which we divide a population into nonoverlapping subpopulations and then select a random sample from each subpopulation (stratum). (page 287)

systematic sample: A sample taken by moving systematically through the population. For instance, we might randomly select one of the first 200 population units and then systematically sample every 200th population unit thereafter. (page 289)

***t* distribution:** A commonly used continuous probability distribution that is described by a distribution curve similar to a normal curve. The *t* curve is symmetrical about zero and is more spread out than a standard normal curve. (page 264)

***t* point, t_α:** The point on the horizontal axis under a t curve that gives a right-hand tail area equal to α. (page 264)

***t* table:** A table of t point values listed according to the area in the tail of the t curve and according to values of the degrees of freedom. (pages 264–266)

undercoverage: A situation in sampling in which some groups of population units are underrepresented. (page 289)

Important Formulas

A large sample confidence interval for a population mean μ: page 259
A small sample confidence interval for a population mean μ: page 266
Sample size when estimating μ: page 273
A large sample confidence interval for a population proportion p: page 276
Sample size when estimating p: page 278
Estimation of a mean and a total for a finite population: page 284
Estimation of a proportion and a total for a finite population: page 285

Supplementary Exercises

7.70 In an article in the *Journal of Accounting Research,* Ashton, Willingham, and Elliott studied audit delay (the length of time from a company's fiscal year-end to the date of the auditor's report) for industrial and financial companies. In the study, a random sample of 250 industrial companies yielded a mean audit delay of 68.04 days with a standard deviation of 35.72 days, while a random sample of 238 financial companies yielded a mean audit delay of 56.74 days with a standard deviation of 34.87 days. Use these sample results to do the following:

a Calculate a 95 percent confidence interval for the mean audit delay for all industrial companies.

b Calculate a 95 percent confidence interval for the mean audit delay for all financial companies.

c By comparing the 95 percent confidence intervals you calculated in parts *a* and *b*, is there strong evidence that the mean audit delay for financial companies is shorter than the mean audit delay for industrial companies? Explain.

7.71 In an article in *Accounting and Business Research,* Beattie and Jones investigate the use and abuse of graphic presentations in the annual reports of United Kingdom firms. The authors found that 65 percent of the sampled companies graph at least one key financial variable, but that 30 percent of the graphics are materially distorted (nonzero vertical axis, exaggerated trend, or the like). Results for U.S. firms have been found to be similar.

a Suppose that in a random sample of 465 graphics from the annual reports of United Kingdom firms, 142 of the graphics are found to be distorted. Find a point estimate of and a 95 percent confidence interval for the proportion of U.K. annual report graphics that are distorted.

b Based on this interval, can we be 95 percent confident that more than 25 percent of all graphics appearing in the annual reports of U.K. firms are distorted? Explain. Does this suggest that auditors should understand proper graphing methods?

c Determine the sample size needed in order to be 95 percent confident that $\hat{p}$, the sample proportion of U.K. annual report graphics that are distorted, is within .03 of p, the true proportion of U.K. annual report graphics that are distorted.

7.72 On January 4, 2000, the Gallup Organization released the results of a poll dealing with the likelihood of computer-related Y2K problems and the possibility of terrorist attacks during the New Year's holiday at the turn of the century.[5] The survey results were based on telephone interviews with a randomly selected national sample of 622 adults, 18 years and older, conducted December 28, 1999.

a The Gallup poll found that 61 percent of the respondents believed that one or more terrorist attacks were likely to happen on the New Year's holiday. Based on this finding, calculate a 95 percent confidence interval for the proportion of all U.S. adults who believed that one or more terrorist attacks were likely to happen on the 2000 New Year's holiday. Based on this interval, is it reasonable to conclude that fewer than two-thirds of all U.S. adults believed that one or more terrorist attacks were likely?

[5] **Source:** World Wide Web, http://www.gallup.com/poll/releases/, The Gallup Organization, January 4, 2000.

b In explaining its survey methods, Gallup states the following: "For results based on this sample, one can say with 95 percent confidence that the maximum error attributable to sampling and other random effects is plus or minus 4 percentage points." Explain how your calculations for part *a* verify that this statement is true.

7.73 The manager of a chain of discount department stores wishes to estimate the total number of erroneous discounts allowed by sales clerks during the last month. A random sample of 200 of the chain's 57,532 transactions for the last month reveals that erroneous discounts were allowed on eight of the transactions. Use this sample information to find a point estimate of and a 95 percent confidence interval for the total number of erroneous discounts allowed during the last month.

7.74 National Motors has equipped the ZX-900 with a new disk brake system. We define the stopping distance for a ZX-900 to be the distance (in feet) required to bring the automobile to a complete stop from a speed of 35 mph under normal driving conditions using this new brake system. In addition, we define μ to be the mean stopping distance of all ZX-900s. One of the ZX-900's major competitors is advertised to achieve a mean stopping distance of 60 feet. National Motors would like to claim in a new advertising campaign that the ZX-900 achieves a shorter mean stopping distance.

Suppose that National Motors randomly selects a sample of $n = 81$ ZX-900s. The company records the stopping distance of each automobile and calculates the mean and standard deviation of the sample of $n = 81$ stopping distances to be $\bar{x} = 57.8$ ft and $s = 6.02$ ft.

a Calculate a 95 percent confidence interval for μ. Can National Motors be 95 percent confident that μ is less than 60 ft? Explain.

b Using the sample of $n = 81$ stopping distances as a preliminary sample, find the sample size necessary to make National Motors 95 percent confident that $\bar{x}$ is within one foot of μ.

7.75 A large construction contractor is building 257 homes, which are in various stages of completion. For tax purposes, the contractor needs to estimate the total dollar value of its inventory due to construction in progress. The contractor randomly selects (without replacement) a sample of 40 of the 257 houses and determines the accumulated costs (the amount of money tied up in inventory) for each sampled house. The contractor finds that the sample mean accumulated cost is $\bar{x} = \$75{,}162.70$ and that the sample standard deviation is $s = \$28{,}865.04$.

a Find a point estimate of and a 99 percent confidence interval for the total accumulated costs (total amount of money tied up in inventory) for all 257 homes that are under construction.

b Using the confidence interval as the basis for your answer, find a reasonable estimate of the largest possible total dollar value of the contractor's inventory due to construction in progress.

7.76 In an article in the *Journal of Retailing,* J. G. Blodgett, D. H. Granbois, and R. G. Walters investigated negative word-of-mouth consumer behavior. In a random sample of 201 consumers, 150 reported that they engaged in negative word-of-mouth behavior (for instance, they vowed never to patronize a retailer again). In addition, the 150 respondents who engaged in such behavior, on average, told 4.88 people about their dissatisfying experience (with a standard deviation equal to 6.11).

a Use these sample results to compute a 95 percent confidence interval for the proportion of all consumers who engage in negative word-of-mouth behavior. On the basis of this interval, would it be reasonable to claim that more than 70 percent of all consumers engage in such behavior? Explain.

b Use the sample results to compute a 95 percent confidence interval for the mean number of people who are told about a dissatisfying experience by consumers who engage in negative word-of-mouth behavior. On the basis of this interval, would it be reasonable to claim that these dissatisfied consumers tell, on average, at least three people about their bad experience? Explain.

7.77 A random sample of 50 perceived age estimates for a model in a cigarette advertisement showed that $\bar{x} = 26.22$ years and that $s = 3.7432$ years. ModelAge

a Use this sample to calculate a 95 percent confidence interval for the population mean age estimate for all viewers of the ad.

b Remembering that the cigarette industry requires that models must appear at least 25 years old, does the confidence interval make us 95 percent confident that the mean perceived age estimate is at least 25? Is the mean perceived age estimate much more than 25? Explain.

7.78 In an article in the *Journal of Management Information Systems,* Mahmood and Mann investigate how information technology (IT) investment relates to company performance. In particular, Mahmood and Mann obtain sample data concerning IT investment for companies that effectively use information systems. Among the variables studied are the company's IT budget as a percentage of company revenue, percentages of the IT budget spent on staff and training, and number of PCs and terminals as a percentage of total employees.

a Suppose a random sample of 15 companies considered to effectively use information systems yields a sample mean IT budget as a percentage of company revenue of $\bar{x} = 2.73$ with a standard deviation of $s = 1.64$. Assuming that IT budget percentages are approximately normally distributed, calculate a 99 percent confidence interval for the mean IT budget as a percentage of company revenue for all firms that effectively use information systems. Does this interval provide evidence that a firm can successfully use information systems with an IT budget that is less than 5 percent of company revenue? Explain.

b Suppose a random sample of 15 companies considered to effectively use information systems yields a sample mean number of PCs and terminals as a percentage of total employees of $\bar{x} = 34.76$ with a standard deviation of $s = 25.37$. Assuming approximate normality, calculate a 99 percent confidence interval for the mean number of PCs and terminals as a percentage of total employees for all firms that effectively use information systems. Why is this interval so wide? What can we do to obtain a narrower (more useful) confidence interval?

7.79 THE INVESTMENT CASE InvestRet

Suppose that random samples of 50 returns for each of the following investment classes give the indicated sample mean and sample standard deviation:

Fixed annuities: $\bar{x} = 7.83\%$, $s = .51\%$

Domestic large cap stocks: $\bar{x} = 13.42\%$, $s = 15.17\%$

Domestic midcap stocks: $\bar{x} = 15.03\%$, $s = 18.44\%$

Domestic small cap stocks: $\bar{x} = 22.51\%$, $s = 21.75\%$

a For each investment class, compute a 95 percent confidence interval for the population mean return.

b Do these intervals suggest that the current mean return for each investment class differs from the historical (1970 to 1994) mean return given in Table 2.16 (page 107)? Explain.

7.80 THE INTERNATIONAL BUSINESS TRAVEL EXPENSE CASE

Recall that the mean and the standard deviation of a random sample of 35 one-day travel expenses in Moscow are $\bar{x} = \$538$ and $s = \$41$. Find a 95 percent confidence interval for the mean, μ, of all one-day travel expenses in Moscow.

7.81 THE UNITED KINGDOM INSURANCE CASE

Assume that the U.K. insurance survey is based on 1,000 randomly selected U.K. households and that 640 of these households spent on life insurance in 1993. Find a 95 percent confidence interval for the proportion, p, of all U.K. households that spent on life insurance in 1993.

7.82 Again consider the situation in which a marketing research firm has compared prices at two supermarket chains, Miller's and Albert's. Recall that the firm has made identical purchases of a standardized shopping plan at several stores owned by each chain with the following results:

Miller's cost: \$119.25, \$121.32, \$122.34, \$120.14, \$122.19, \$123.71, \$121.72, \$122.42, \$123.63, \$122.44

Albert's cost: \$111.19, \$114.88, \$115.11, \$117.02, \$116.89, \$116.62, \$115.38, \$114.40, \$113.91, \$111.87

a Assuming normality, calculate 95 percent confidence intervals for the mean costs of the shopping plan at Miller's and at Albert's.

b Using these confidence intervals as the basis for your answer, is there evidence that the mean costs of the shopping plan at the two supermarket chains differ? Explain.

7.83 Internet Exercise

What is the average selling price of a home? The Data and Story Library (DASL) contains data, including the sale price, for a random sample of 117 homes sold in Albuquerque, New Mexico. Go to the DASL website (http://lib.stat.cmu.edu/DASL/) and retrieve the home price data set (http://lib.stat.cmu.edu/DASL/Datafiles/homedat.html.) Use MINITAB, Excel, or MegaStat to produce appropriate graphical (histogram, stem-and-leaf, box plot) and numerical summaries of the price data. Identify, from your numerical summaries, the sample mean and standard deviation. Use these summaries to construct a 99% confidence interval for μ, the mean sale price. Use statistical software (MINITAB, Excel, or MegaStat) to compute a 99% confidence interval for μ. Do the results of your hand calculations agree with those from your statistical software?

Technical note: There are many ways to capture the home price data from the DASL site. One simple way is to select just the rows containing the data values (and not the labels), copy, paste directly into an Excel or MINITAB worksheet, add your own variable labels, and save the resulting worksheet. It is possible to copy the variable labels from DASL as well, but the differences in alignment and the intervening blank line add to the difficulty. AlbHome

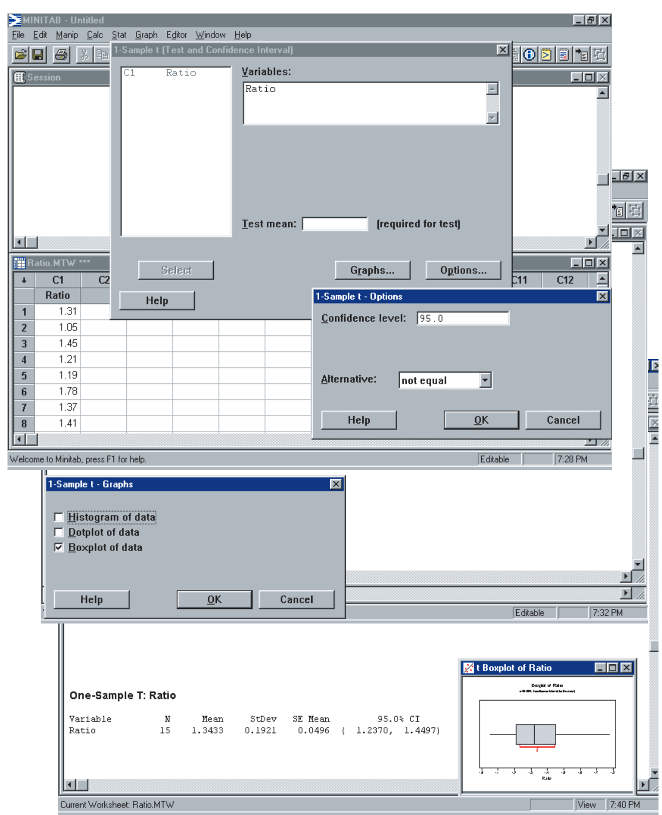
Appendix 7.1 ■ Confidence Intervals Using MINITAB

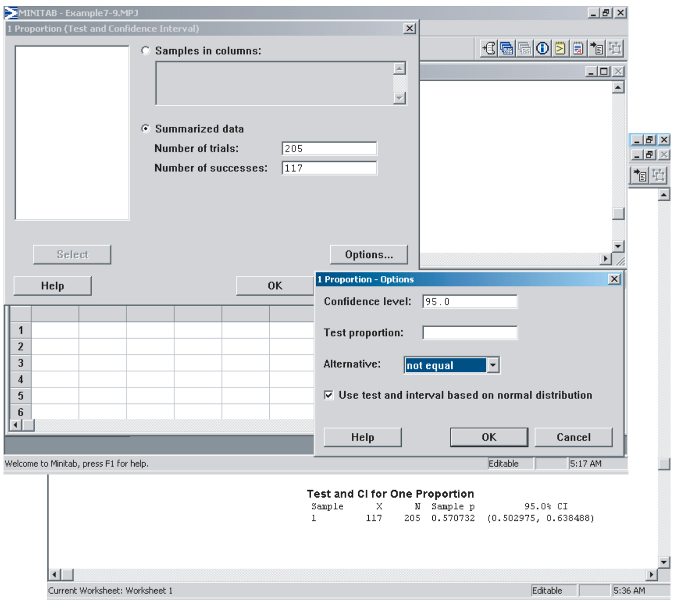
The instruction block in this section begins by describing the entry of data into the MINITAB Data window. Alternatively, the data may be loaded directly from the data disk included with the text. The appropriate data file name is given at the top of the instruction block. Please refer to Appendix 1.1 for further information about entering data, and saving and printing results.

Confidence interval for a population mean in Figure 7.12(a) on page 269 (data file: Ratio.mtw):

- In the Data window, enter the debt-to-equity ratio data from Example 7.4 (page 267) into a single column called Ratio.
- Select **Stat : Basic Statistics : 1-Sample t.**
- In the "1-Sample t (Test and Confidence Interval)" dialog box, select the Ratio variable into the Variables box.
- Click the Options . . . button and enter 95.0 into the Confidence level box.
- Select "not equal" in the Alternative box, and click OK in the "1-Sample t Options" dialog box.
- To produce a box plot of the data with a graphical representation of the confidence interval—click the Graphs . . . button, check the "Boxplot of data" check box, and click OK in the "1-Sample t Graphs" dialog box.
- Click OK in "1-Sample t (Test and Confidence Interval)" dialog box.
- The confidence interval is given in the session window, and the box plot appears in a graphics window.

A "1-Sample Z" interval is also available in MINITAB under Basic Statistics. It requires a user-specified value of the population standard deviation, which is rarely known.



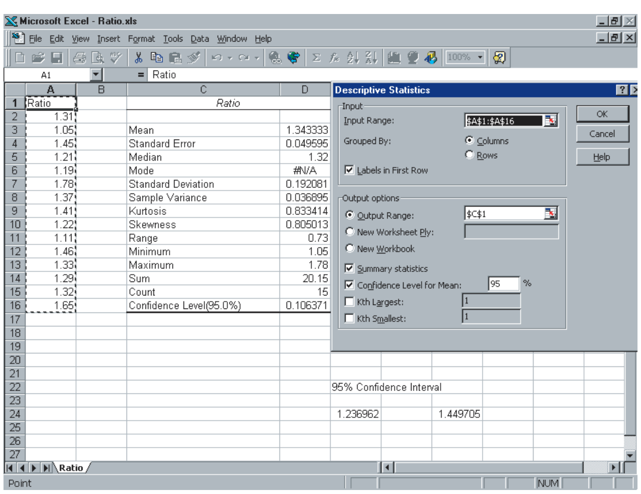


Confidence interval for a population proportion in the marketing ethics situation of Example 7.9 on page 278.

- Select **Stat : Basic Statistics : 1 Proportion.**
- In the "1 Proportion (Test and Confidence Interval)" dialog box, select "Summarized data."
- Enter the number of trials (here equal to 205) and the number of successes (here equal to 117) into the appropriate boxes.
- Click on the Options button.
- In the "1 Proportion—Options" dialog box, enter the desired level of confidence (here 95.0) into the "Confidence level" box.
- Click OK in the "1 Proportion—Options" dialog box.
- Click OK in the "1 Proportion (Test and Confidence Interval)" dialog box.

Appendix 7.2 Confidence Intervals Using Excel

The instruction block in this section begins by describing the entry of data into an Excel spreadsheet. Alternatively, the data may be loaded directly from the data disk included with the text. The appropriate data file name is given at the top of the instruction block. Please refer to Appendix 1.2 for further information about entering data, and saving and printing results.



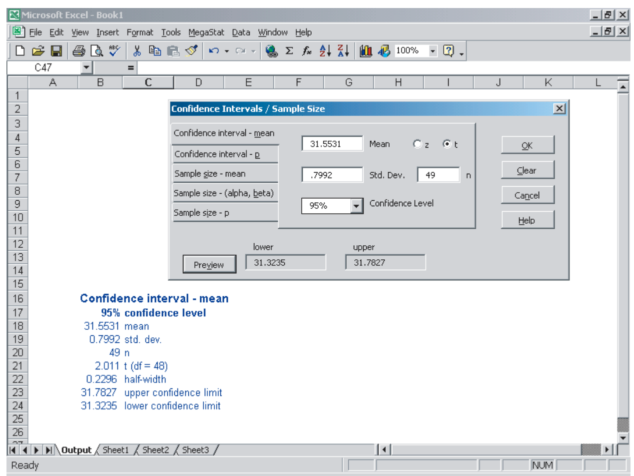
Confidence interval for a population mean in Figure 7.11(b) on page 269 (data file: Ratio.xls):

- Enter the debt-to-equity ratio data from Example 7.4 (page 267) into cells A2 to A16 with the label Ratio in cell A1.
- Select **Tools : Data Analysis : Descriptive Statistics.**
- Click OK in the Data Analysis dialog box.
- In the Descriptive Statistics dialog box, enter A1.A16 into the Input Range box.
- Click the "Labels in First Row" check box.
- Click Output Range and enter C1 in the Output Range box.
- Click the Summary Statistics and "Confidence Level for Mean" check boxes. This produces a *t*-based error bound for a confidence interval for both large (>=30) and small (<30) samples.
- Type 95 in the Confidence Level box.
- Click OK in the Descriptive Statistics dialog box.
- A descriptive statistics summary will appear in cells C3 through D16. Drag the column C border to uncover complete labels for all of the descriptive statistics.

- Type the heading "95% Confidence Interval" into cells E22 to G22.
- Compute the lower bound of the interval by typing the formula =D3-D16 into cell E24. This subtracts the half-length of the interval (labeled "Confidence Level (95%)") from the sample mean.
- Compute the upper bound of the interval by typing the formula =D3+D16 into cell G24.

Appendix 7.3 ■ Confidence Intervals Using MegaStat

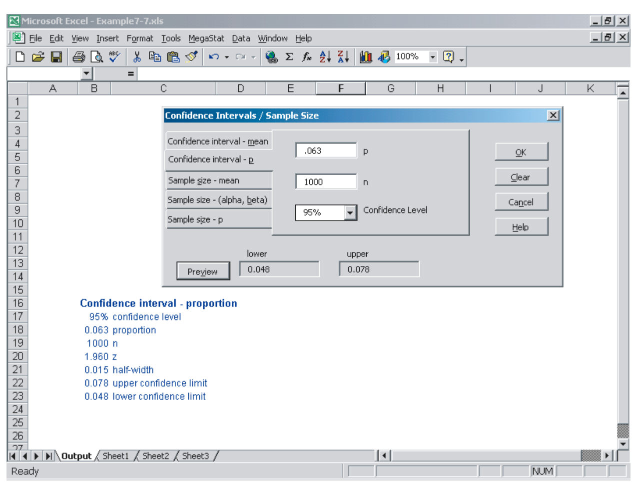
Confidence interval for the population mean gas mileage similar to the output in Figure 7.11(a) on page 269.



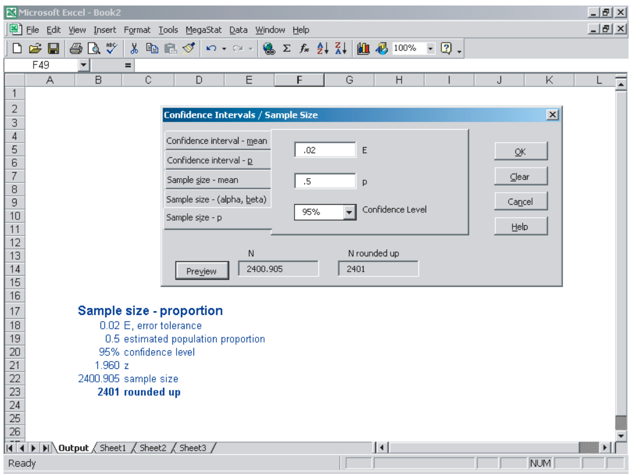
- Select **MegaStat : Confidence Intervals / Sample Size.**
- In the "Confidence Intervals / Sample Size" dialog box, click on the "Confidence Interval—mean" tab.
- Enter the sample mean (here equal to 31.5531) into the Mean box.
- Enter the sample standard deviation (here equal to .7992) into the "Std Dev" box.
- Enter the sample size (here equal to 49) into the "n" box.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Select a *t*-based or *z*-based interval by clicking on "t" or "z". Here we request a *t*-based interval.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.

If we wish to use raw data, a *t*-based confidence interval for a population mean can be obtained by using the Descriptive Statistics dialog box—see page 117.

Confidence interval for a population proportion in the cheese spread situation of Example 7.7 on pages 276–277.



- In the "Confidence Intervals / Sample Size" dialog box, click on the "Confidence Interval—p" tab.
- Enter the sample proportion (here equal to .063) into the "p" box.
- Enter the sample size (here equal to 1,000) into the "n" box.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.



Sample size determination for a population proportion in Figure 7.17 on page 279.

- In the "Confidence Intervals / Sample Size" dialog box, click on the "Sample size—p" tab.
- Enter the desired error bound (here equal to 0.02) into the "E" box and enter an estimate of the population proportion into the "p" box.
- Select a level of confidence from the pull-down menu or type a desired percentage.
- Click OK in the "Confidence Intervals / Sample Size" dialog box.

Sample size determination for a population mean problem is done by clicking on the "Sample Size—mean" tab. Then enter a desired error bound, an estimate of the population standard deviation, and the desired level of confidence. Click OK.