UNIVERSITY OF NEWCASTLE UPON TYNE

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 2
2003/2004

MAS357

Time Series and Forecasting

Time allowed: 1 hour 30 minutes

Credit will be given for ALL answers to questions in Section A, and for the best TWO answers to questions in Section B. No credit will be given for other answers and students are strongly advised not to spend time producing answers for which they will receive no credit.

Marks allocated to each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are THREE questions in Section A and THREE questions in Section B.

SECTION A

A1. (a) Give the definitions of the autocovariance and autocorrelation of a time series model.

(b) Calculate the autocovariance and autocorrelation function for the time series

$$Y_t = 3 + Z_t + \frac{1}{2}Z_{t-1} + \frac{1}{5}Z_{t-2},$$

where $\{Z_t\}$ are independent $N(0,1)$ random variables.

[14 marks]

A2. Consider the time series

$$Y_t = \mu + \phi Y_{t-1} + Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \theta_3 Z_{t-3},$$

where $\{Z_t\}$ are independent $N(0,1)$ random variables, μ is the constant of the model and ϕ, θ_1, θ_2 and θ_3 are the parameters of the model.

(a) Give the full name and abbreviation of the above time series model. Define the backward shift operator B and write the above time series in the backward shift operator form

$$\Phi(B)Y_t = \mu + \Theta(B)Z_t,$$

by specifying $\Phi(B)$ and $\Theta(B)$.

(b) Give the one-step forecast y_{T+1} when the above model is fitted to a time series of length T.

[12 marks]

A3. (a) Give definitions for the information set D_T and the k-step forecast distribution in the context of the local level model.

(b) Given a time series of length T, give the definition of the correlogram and explain how the correlogram can be used in practical data analysis.

(c) Explain when a time series of length T can be modelled with regression techniques and when the modeller has to resort to time series techniques. Give a simple graphical example.

[14 marks]

SECTION B

B4. Consider the following local level model

$$Y_t = \mu_t + \nu_t, \qquad \nu_t \sim N(0,1),$$
$$\mu_t = \mu_{t-1} + \omega_t, \qquad \omega_t \sim N(0,1), \qquad (1)$$

defined for $t = 1, 2, \ldots, 220$.

(a) Show that this model can be written as an ARMA(1,1) model with white noise model. Is this model stationary? Give brief explanations.

(b) Using the system equation (1) express the level μ_{t+20} in terms of μ_t and $\omega_{t+1}, \omega_{t+2}, \ldots, \omega_{t+20}$, for any $t = 1, 2, \ldots, 200$.

(c) Suppose that at time $t = 200$, we know that $\mu_{200}|D_{200} \sim N(5, 50)$. Calculate the expectation and the variance of $Y_{220}|D_{200}$.

(d) Give the definition of the k-step ahead error e_{t+k} for each $t = 1, 2, \ldots, 220 - k$ where the positive integer k satisfies $1 \leq k \leq 119$. Using (b) and (c), or otherwise, derive the distribution of the 20-step ahead error $e_{220}|D_{200}$.

[30 marks]

B5. Consider the following AR(2) model

$$Y_t = \mu + abY_{t-1} + ab^2Y_{t-2} + \nu_t, \qquad \nu_t \sim iN(0,1),$$

where a and b are positive real numbers and $t > 0$.

(a) Find an upper bound of b so that the time series $\{Y_t\}$ is stationary for any $a > 0$.

(b) (i) Write down and briefly discuss the Yule-Walker equations for the above time series model.

(ii) Let $a = 1$ and $b^2 = 0.2$. Verify that the time series $\{Y_t\}$ is stationary. If the autocorrelation function at lag 3 is $\rho_3 = 0.4$ calculate ρ_1 the autocorrelation function at lag 1.

Question B5 continued on next page

(c) If $\mu = 2.8$ and a and b are as in (b), give the one-step and two-step forecasts when the above model is fitted to a time series of length 100 with observed values $y_{99} = 7.3$ and $y_{100} = 7.1$. If $y_{101} = 8$ and $y_{102} = 6.9$ calculate the respective one and two-step ahead errors.

[30 marks]

B6. (a) Consider the MA(1) time series

$$Y_t = Z_t + \frac{1}{4} Z_{t-1}$$

where $\{Z_t\}$ are independent $N(0, 8)$ random variables and $t > 1$. Given that the autocovariance function of the above time series is

$$\gamma_k = \begin{cases} \frac{1}{4} & k = 1 \\ 0 & k \geq 2 \end{cases}$$

determine the autocorrelation function and find another MA(1) model having the same autocorrelation function with the above time series model.

(b) Suppose the time series $\{Y_t\}$ is stationary and has autocovariance function γ_k. A new stationary time series $\{X_t\}$ is defined by $X_t = Y_t - Y_{t-1}$, for all $t \geq 2$.

(i) Express the autocovariance function γ'_k of $\{X_t\}$ in terms of the autocovariance function of $\{Y_t\}$.

(ii) Find the autocorrelation function of $\{X_t\}$ when $\gamma_k = (1/3)^k$, $k = 0, 1, 2, \ldots$

(c) 200 observations $y_1, y_2, \ldots, y_{200}$ of a time series $\{Y_t\}$ gave sample autocorrelation at lags 1,2,3,4,5 as $r_1 = 0.427$, $r_2 = 0.475$, $r_3 = 0.169$, $r_4 = 0.110$ and $r_5 = 0.005$. If μ is the theoretical expectation of the series $\{Y_t\}$, then

(i) Give an approximate confidence interval for the theoretical autocorrelation function ρ_k using a time series of length $T > 0$. Determine the approximate confidence interval for the theoretical autocorrelation function ρ_k for the time series of (c).

(ii) Based on the sample statistics given in (c), is it reasonable to suppose that $Y_t - \mu$ is white noise? If not suggest a possible candidate time series model for $\{Y_t\}$. Give explanations.

[30 marks]