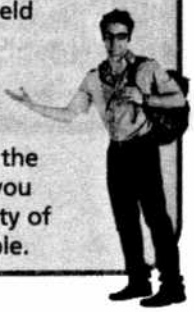
Increasing Sample Size to Increase Precision

One way to increase the precision of the confidence interval without decreasing the level of confidence is to increase the sample size.

Insight

The reason for using 0.5 as values for $\hat{p}$ and $\hat{q}$ when no preliminary estimate is available is that these values yield a maximum value for the product $\hat{p}\hat{q} = \hat{p}(1 - \hat{p})$. In other words, if you don't estimate the values of $\hat{p}$ and $\hat{q}$, you must pay the penalty of using a larger sample.



FINDING A MINIMUM SAMPLE SIZE TO ESTIMATE p

Given a c-confidence level and a margin of error E, the minimum sample size n needed to estimate p is

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2.$$

This formula assumes that you have a preliminary estimate for $\hat{p}$ and $\hat{q}$. If not, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

EXAMPLE 4

Determining a Minimum Sample Size

You are running a political campaign and wish to estimate, with 95% confidence, the proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the true population. Find the minimum sample size needed if (1) no preliminary estimate is available and (2) a preliminary estimate gives $\hat{p} = 0.31$. Compare your results.

Solution

1. Because you do not have a preliminary estimate for $\hat{p}$, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$. Using $z_c = 1.96$ and $E = 0.03$, you can solve for n.

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \approx 1067.11$$

   Because n is a decimal, round up to the nearest whole number, 1068.

2. You have a preliminary estimate of $\hat{p} = 0.31$. So, $\hat{q} = 0.69$. Using $z_c = 1.96$ and $E = 0.03$, you can solve for n.

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.31)(0.69)\left(\frac{1.96}{0.03}\right)^2 \approx 913.02$$

   Because n is a decimal, round up to the nearest whole number, 914.

Interpretation With no preliminary estimate, the minimum sample size should be at least 1068 voters. With a preliminary estimate of $\hat{p} = 0.31$, the sample size should be at least 914 voters. So, you would need a larger sample size if no preliminary estimate is available.

Try It Yourself 4

You wish to estimate, with 90% confidence and within 2% of the true population, the proportion of males age 20 to 34 who have high blood pressure. Find the minimum sample size needed if (1) no preliminary estimate is available and (2) a previous survey found that 6.4% of males in this age group had high blood pressure. *(Source: National Center for Health Statistics)*

a. *Identify* $\hat{p}$, $\hat{q}$, z_c, and E. If $\hat{p}$ is unknown, use 0.5.
b. Use $\hat{p}$, $\hat{q}$, z_c, and E to *find* the minimum sample size n.
c. *Determine* how many males should be included in the sample.

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