c. Suppose that capital input (number of buses) is decreased by 3 percent next year (when certain older buses are taken out of service). Assuming that the other inputs are held constant, determine the approximate percentage change in output.

d. What type of returns to scale appears to characterize this bus transportation system? (Ignore the issue of statistical significance.)

e. Discuss some of the methodological and measurement problems one might encounter in using time-series data to estimate the parameters of this model.

12. *Extension of the Cobb-Douglas Production Function*—The Cobb-Douglas production function (Equation 7.16) can be shown to be a special case of a larger class of production functions having the following mathematical form:[12]

$$Q = \gamma[\partial K^{-\rho} + (1 - \partial)L^{-\rho}]^{-\nu/\rho}$$

where γ is an efficiency parameter that shows the output resulting from given quantities of inputs; ∂ is a distribution parameter ($0 \leq \partial \leq 1$) that indicates the division of factor income between capital and labor; ρ is a substitution parameter that is a measure of substitutability of capital for labor (or vice versa) in the production process; and ν is a scale parameter ($\nu > 0$) that indicates the type of returns to scale (increasing, constant, or decreasing). Show that when $\nu = 1$, this function exhibits constant returns to scale. [*Hint:* Increase capital K and labor L each by a factor of λ, or $K^* = (\lambda)K$ and $L^* = (\lambda)L$, and show that output Q also increases by a factor of λ, or $Q^* = (\lambda)(Q)$.]

13. Lobo Lighting Corporation currently employs 100 unskilled laborers, 80 factory technicians, 30 skilled machinists, and 40 skilled electricians. Lobo feels that the marginal product of the last unskilled laborer is 400 lights per week, the marginal product of the last factory technician is 450 lights per week, the marginal product of the last skilled machinist is 550 lights per week, and the marginal product of the last skilled electrician is 600 lights per week. Unskilled laborers earn $400 per week, factory technicians earn $500 per week, machinists earn $700 per week, and electricians earn $750 per week.

Is Lobo using the lowest cost combination of workers to produce its targeted output? If not, what recommendations can you make to assist the company?

CASE EXERCISE

THE PRODUCTION FUNCTION FOR WILSON COMPANY

Economists at the Wilson Company are interested in developing a production function for fertilizer plants. They collected data on 15 different plants that produce fertilizer (see the following page).

QUESTIONS

1. Estimate the Cobb-Douglas production function $Q = \alpha L^{\beta_1} K^{\beta_2}$, where Q = output; L = labor input; K = capital input; and α, β_1, and β_2 are the parameters to be estimated.
2. Test whether the coefficients of capital and labor are statistically significant.
3. Determine the percentage of the variation in output that is "explained" by the regression equation.

[12] See R. G. Chambers, *Applied Production Analysis* (Cambridge: Cambridge University Press, 1988).

4. Determine the labor and capital estimated parameters and give an economic interpretation of each value.

5. Determine whether this production function exhibits increasing, decreasing, or constant returns to scale. (Ignore the issue of statistical significance.)

Plant	Output (000 Tons)	Capital ($000)	Labor (000 Worker Hours)
1	605.3	18,891	700.2
2	566.1	19,201	651.8
3	647.1	20,655	822.9
4	523.7	15,082	650.3
5	712.3	20,300	859.0
6	487.5	16,079	613.0
7	761.6	24,194	851.3
8	442.5	11,504	655.4
9	821.1	25,970	900.6
10	397.8	10,127	550.4
11	896.7	25,622	842.2
12	359.3	12,477	540.5
13	979.1	24,002	949.4
14	331.7	8,042	575.7
15	1064.9	23,972	925.8