UNIVERSITY OF NEWCASTLE UPON TYNE

SCHOOL OF MATHEMATICS & STATISTICS

SEMESTER 1
2003/2004

MAS314/PHY376

Relativity

Time allowed: 1 hour 30 minutes

Credit will be given for ALL answers to questions in Section A, and for the best TWO answers to questions in Section B. No credit will be given for other answers and students are strongly advised not to spend time producing answers for which they will receive no credit.

Marks allocated to each question are indicated. However you are advised that marks indicate the relative weight of individual questions, they do not correspond directly to marks on the University scale.

There are NINE questions in Section A and THREE questions in Section B.

You may use the following values of the fundamental constants for this examination where appropriate.

$c = 3.00 \times 10^{8}\ m\,s^{-1}$
$G = 6.67 \times 10^{-11}\ N\,m^{2}\,kg^{-2}$
$h = 6.63 \times 10^{-34}\ Js$
$\epsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$
$\epsilon_0\mu_0 = c^{-2}$
Mass of the sun $= 1.99 \times 10^{30}\ kg$
Mass of the earth $= 5.98 \times 10^{24}\ kg$
Mass of the electron $= 0.511\ MeV/c^{2}$
Mass of the proton $= 938\ MeV/c^{2}$

SECTION A

A1. Inertial observer $\mathcal{O}'$ moves at speed v in the negative z-direction with respect to inertial observer $\mathcal{O}$. Write down the relationship between the sets of coordinates used by the two different observers.

[4 marks]

A2. A rocket moves at speed $4c/5$ towards the earth. It fires a missile at a speed of $3c/5$ opposite to its direction of motion as measured on the rocket. With what speed does the missile strike the earth?

[4 marks]

A3. If $\mathbf{A} = (\lambda, 1, 2, 2)$ is a future-pointing timelike 4-vector, what is the maximum information you know about λ?

[4 marks]

A4. A particle is measured to have a kinetic energy that is four times its rest mass energy. How fast is the particle moving? Express the speed as a fraction of c.

[4 marks]

A5. Write down the transformations that relate the electric field $\vec{E}$ and the magnetic field $\vec{B}$ in two different inertial frames. Use your result to show that $\vec{E} \cdot \vec{B}$ is the same for all inertial observers. You may choose the motion to be in the x-direction if you wish.

[8 marks]

A6. Define the Schwarzschild radius of a black hole and give a brief description of its significance.

[4 marks]

A7. Give a very brief description of two observational tests of general relativity.

[4 marks]

A8. An inertial observer sees a particle moving at a speed $3c/5$. It then is seen to decay into two photons. Draw an accurate spacetime diagram to illustrate the situation before and after the decay takes place.

[4 marks]

A9. What energies, expressed in terms of the rest mass energy of the initial particle, would the observer in question **A8** measure for the two photons?

[4 marks]

SECTION B

B10. (a) Define what is meant by a 4-vector $\mathbf{A}$. [4 marks]

(b) If $\mathbf{A}$ and $\mathbf{B}$ are both 4-vectors, prove that $\mathbf{A} \cdot \mathbf{B}$ is invariant under a Lorentz transformation. [4 marks]

(c) If $\mathbf{T}$ is a timelike 4-vector, prove that it is possible to perform a Lorentz transformation to a new inertial frame so that $\mathbf{T}$ has only a time component in this new frame. [14 marks]

(d) If $\mathbf{T}$ is a timelike 4-vector and $\mathbf{N}$ is a null 4-vector, is it ever possible for $\mathbf{T} + \mathbf{N}$ to be a null 4-vector? [8 marks]

[30 marks]

B11. (a) What is meant by the threshold energy in a particle reaction? If the threshold energy in a particle reaction holds, what can you say about the motion of the particles in the centre of momentum frame? What does this imply about the motion of the particles in the original laboratory frame? [6 marks]

(b) Two photons of energies E_1 and E_2 collide with their incident directions of motion making an angle θ with respect to each other. The collision annihilates the two photons and produces two new particles of equal rest mass m. Show that at threshold the condition

$$E_1 E_2 \sin^2(\theta/2) = m^2 c^4$$

holds. [24 marks]

[30 marks]

B12. (a) Write down the line element that describes the spacetime geometry outside the event horizon of a spherically symmetric black hole of mass M. [4 marks]

(b) What equation describes the radial infall of a light ray into a spherically symmetric black hole? [4 marks]

(c) At time $t = 0$ a light signal is sent from a fixed position at $r = 4r_S$ radially outwards to a distant fixed observer outside a Schwarzschild black hole where r_S is the Schwarzschild radius. A second light signal is sent from $r = 2r_S$ radially outwards towards the same observer also at the time $t = 0$. If the difference in travel times for these two signals is measured to be $3.05 \times 10^{-2}s$ by the observer, deduce the mass of the black hole in terms of the mass of the sun. [22 marks]

[30 marks]