

Group 1 Solutions 15, 25, 27, 30, 31

Applications of Normal Distributions SECTION 6-3 155

11. The z score with 0.95 above is the z score with 0.05 below it; $z = -1.645$ [bottom of table].

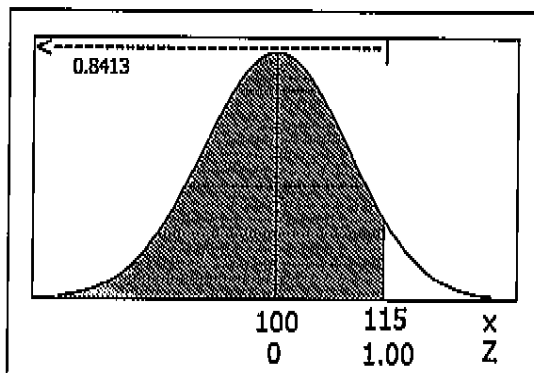
$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (-1.645)(15) \\ &= 100 - 24.675 = 75.325, \text{ rounded to } 75.3 \\ \text{Excel: } &\text{NORMINV}(0.05, 100, 15) = 75.3 \end{aligned}$$

12. The z score with 0.99 below it is $z = 2.33$ [closest entry 0.9901].

$$\begin{aligned} x &= \mu + z\sigma \\ &= 100 + (2.33)(15) \\ &= 100 + 34.95 = 134.95, \text{ rounded to } 135.0 \\ \text{Excel: } &\text{NORMINV}(0.99, 100, 15) = 135.0 \end{aligned}$$

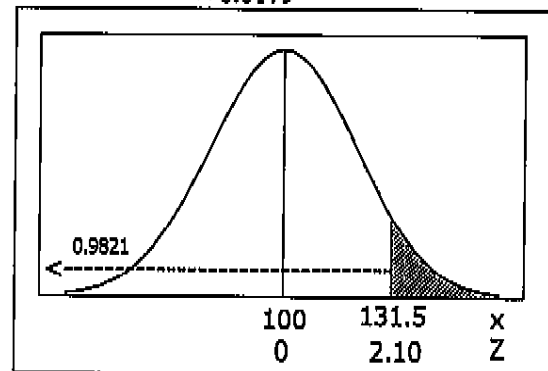
13. normal distribution: $\mu = 100$ and $\sigma = 15$
 $P(x < 115)$
 $= P(z < 1.00)$
 $= 0.8413$

Excel: $\text{NORMDIST}(115, 100, 15, 1) = 0.8413$



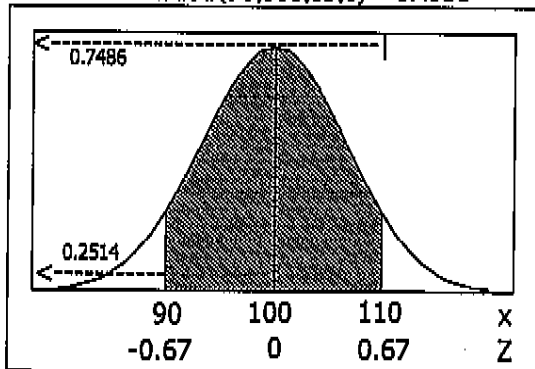
14. normal distribution: $\mu = 100$ and $\sigma = 15$
 $P(x > 131.5)$
 $= P(z > 2.10)$
 $= 1 - 0.9821$
 $= 0.0179$

Excel: $1 - \text{NORMDIST}(131.5, 100, 15, 1) = 0.0179$



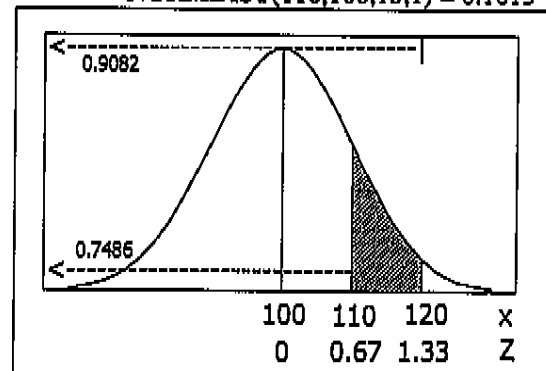
15. normal distribution: $\mu = 100$ and $\sigma = 15$
 $P(90 < x < 110) = P(-0.67 < z < 0.67)$
 $= 0.7486 - 0.2514$
 $= 0.4972$

Excel: $\text{NORMDIST}(110, 100, 15, 1) - \text{NORMDIST}(90, 100, 15, 1) = 0.4950$



16. normal distribution: $\mu = 100$ and $\sigma = 15$
 $P(110 < x < 120) = P(0.67 < z < 1.33)$
 $= 0.9082 - 0.7486$
 $= 0.1596$

Excel: $\text{NORMDIST}(120, 100, 15, 1) - \text{NORMDIST}(110, 100, 15, 1) = 0.1613$

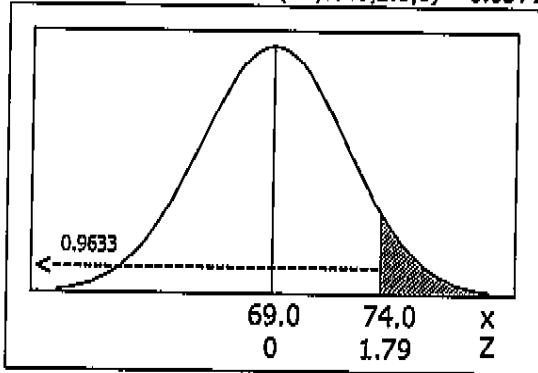


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23. a. normal distribution: $\mu = 69.0, \sigma = 2.8$

$$\begin{aligned} P(x > 74) &= P(z > 1.79) \\ &= 1 - 0.9633 \\ &= 0.0367 \text{ or } 3.67\% \end{aligned}$$

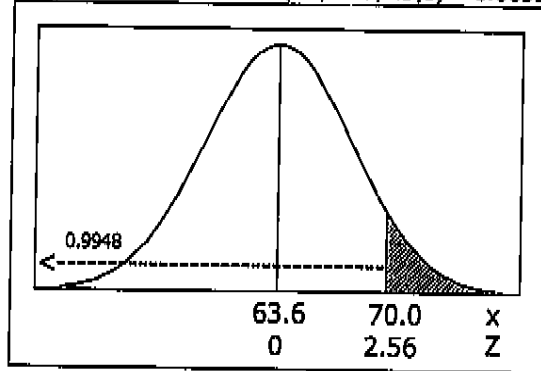
Excel: $1 - \text{NORMDIST}(74, 69.0, 2.8, 1) = 0.0371$



b. normal distribution: $\mu = 63.6, \sigma = 2.5$

$$\begin{aligned} P(x > 70) &= P(z > 2.56) \\ &= 1 - 0.9948 \\ &= 0.0052 \text{ or } 0.52\% \end{aligned}$$

Excel: $1 - \text{NORMDIST}(70, 63.6, 2.5, 1) = 0.0052$



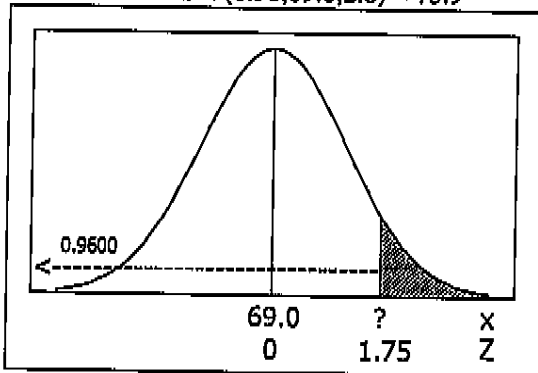
c. No. The requirements are not equally fair for men and women, since the percentage of men that are eligible is so much larger than the percentage of women who are eligible.

24. a. normal distribution: $\mu = 69.0, \sigma = 2.8$

For the tallest 4%, $A = 0.9600$ [0.9599] and $z = 1.75$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 69.0 + (1.75)(2.8) \\ &= 69.0 + 4.9 \\ &= 73.9 \text{ inches} \end{aligned}$$

Excel: $\text{NORMINV}(0.96, 69.0, 2.8) = 73.9$

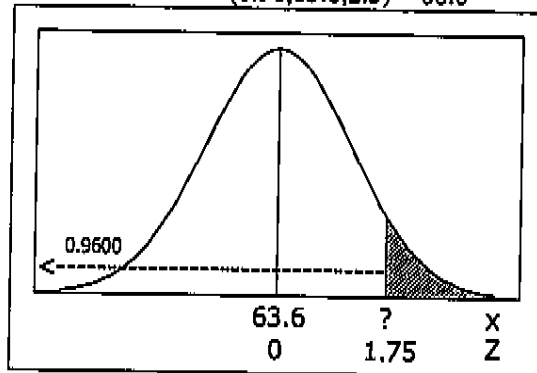


b. normal distribution: $\mu = 63.6, \sigma = 2.5$

For the tallest 4%, $A = 0.9600$ [0.9599] and $z = 1.75$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 63.6 + (1.75)(2.5) \\ &= 63.6 + 4.4 \\ &= 68.0 \text{ inches} \end{aligned}$$

Excel: $\text{NORMINV}(0.96, 63.6, 2.5) = 68.0$

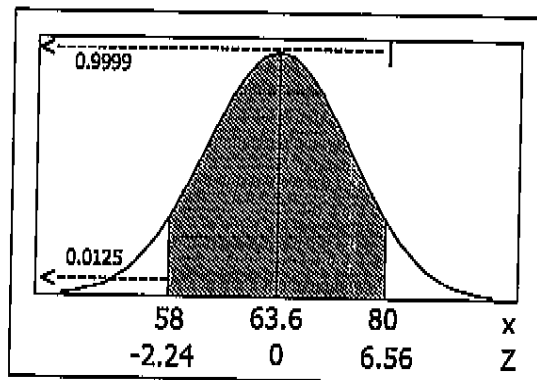


25. normal distribution: $\mu = 63.6, \sigma = 2.5$

$$\begin{aligned} \text{a. } P(58 < x < 80) &= P(-2.24 < z < 6.56) \\ &= 0.9999 - 0.0125 \\ &= 0.9874 \text{ or } 98.74\% \end{aligned}$$

No. Only $1 - 0.9874 = 0.0126 = 1.26\%$ of the women are not eligible because of the height requirements.

Excel: $\text{NORMDIST}(80, 63.6, 2.5, 1) - \text{NORMDIST}(58, 63.6, 2.5, 1) = 0.9875$

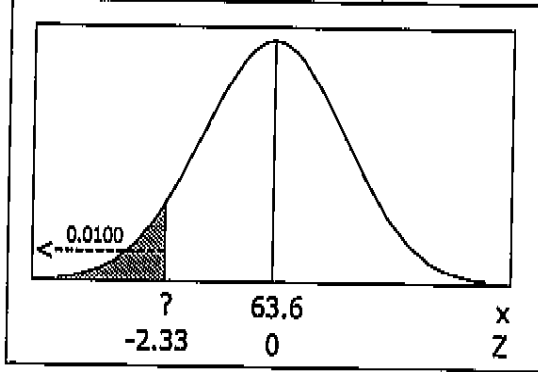


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b. For the shortest 1%, $A = 0.0100$ [0.0099] and $z = -2.33$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 63.6 + (-2.33)(2.5) \\ &= 63.6 - 5.8 \\ &= 57.8 \text{ inches} \end{aligned}$$

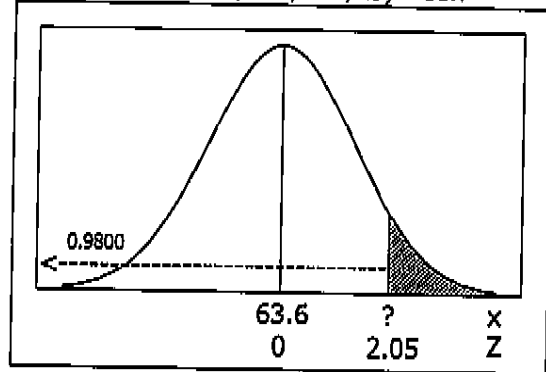
Excel: `NORMINV(0.01,63.6,2.5) = 57.8`



For the tallest 2%, $A = 0.9800$ [0.9798] and $z = 2.05$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 63.6 + (2.05)(2.5) \\ &= 63.6 + 5.1 \\ &= 68.7 \text{ inches} \end{aligned}$$

Excel: `NORMINV(0.98,63.6,2.5) = 68.7`

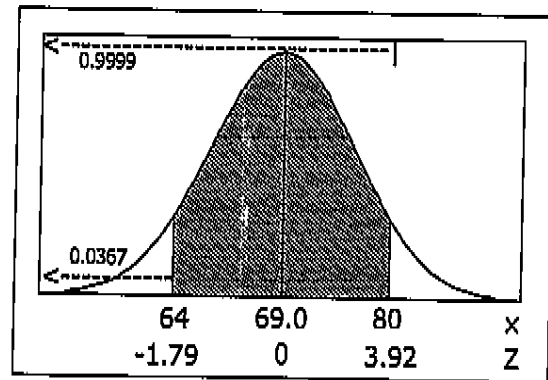


26. normal distribution: $\mu = 69.0$, $\sigma = 2.8$

a. $P(64 < x < 80) = P(-1.79 < z < 3.92)$
 $= 0.9999 - 0.0367$
 $= 0.9632$ or 96.32%

No. Only $1 - 0.9632 = 0.0368 = 3.68\%$ of the men are not eligible because of the height requirements.

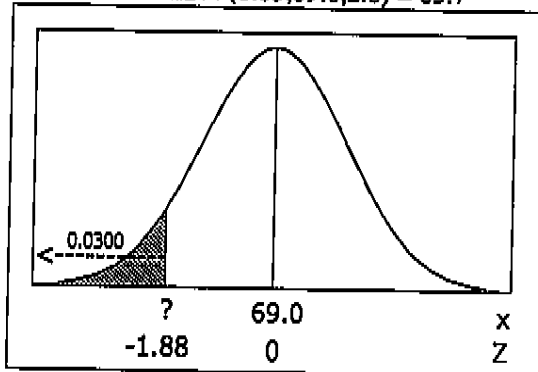
Excel: `NORMDIST(80,69.0,2.8,1)`
`- NORMDIST(64,69.0,2.8,1) = 0.9629`



b. For the shortest 3%, $A = 0.0300$ [0.0301] and $z = -1.88$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 69.0 + (-1.88)(2.8) \\ &= 69.0 - 5.3 \\ &= 63.7 \text{ inches} \end{aligned}$$

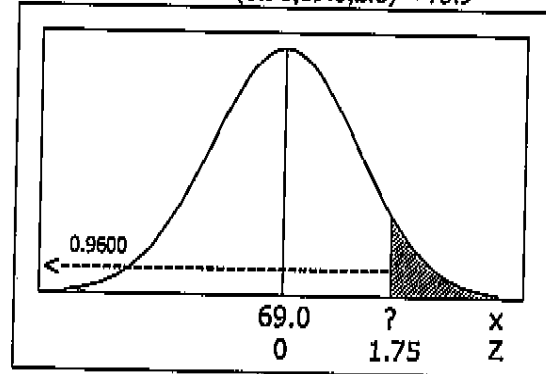
Excel: `NORMINV(0.03,69.0,2.8) = 63.7`



For the tallest 4%, $A = 0.9600$ [0.9599] and $z = 1.75$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 69.0 + (1.75)(2.8) \\ &= 69.0 + 4.9 \\ &= 73.9 \text{ inches} \end{aligned}$$

Excel: `NORMINV(0.96,69.0,2.8) = 73.9`



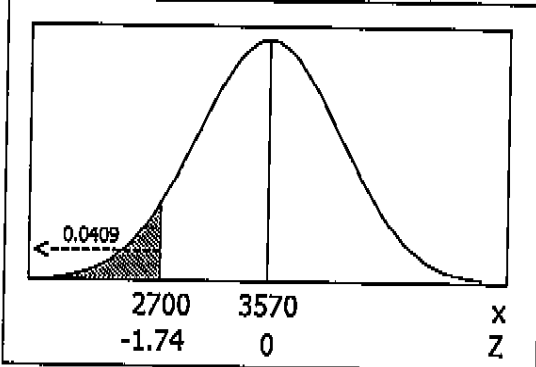
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27. normal distribution: $\mu = 3570$ and $\sigma = 500$

a. $P(x < 2700)$
 $= P(z < -1.74)$
 $= 0.0409$ or 4.09%

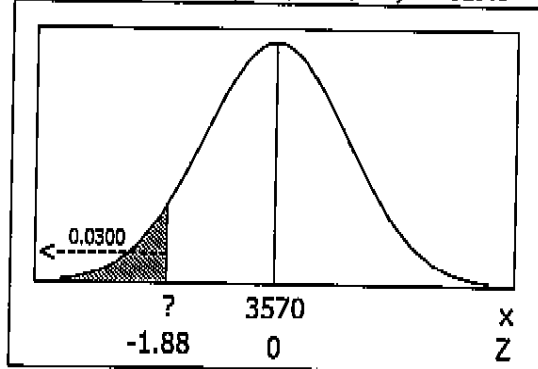
Excel: `NORMDIST(2700,3570,500,1) = 0.0409`



b. For the lightest 3%, $A = 0.0300$ [0.0301] and $z = -1.88$

$x = \mu + z\sigma$
 $= 3570 + (-1.88)(500)$
 $= 3570 - 940 = 2630$ g

Excel: `NORMINV(0.03,3570,500) = 2629.6`



c. Not all babies below a certain birth weight require special treatment. The need for special treatment is determined at least as much by developmental considerations as by weight alone. Also, the birth weight identifying the bottom 3% is not a static figure and would have to be updated periodically – perhaps creating unnecessary uncertainty and inconsistency.

28. normal distribution: $\mu = 172$ and $\sigma = 29$

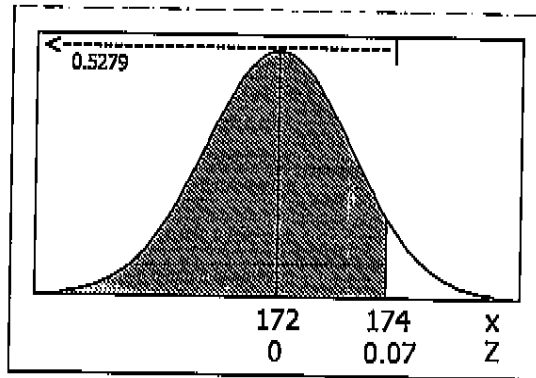
a. $P(x < 174) = P(z < 0.07)$
 $= 0.5279$

Excel: `NORMDIST(174,172,29,1) = 0.5275`

b. $3500/140 = 25$ men

c. $3500/174 = 20$ men

d. Historically, the mean weight of American adults has been increasing over time.

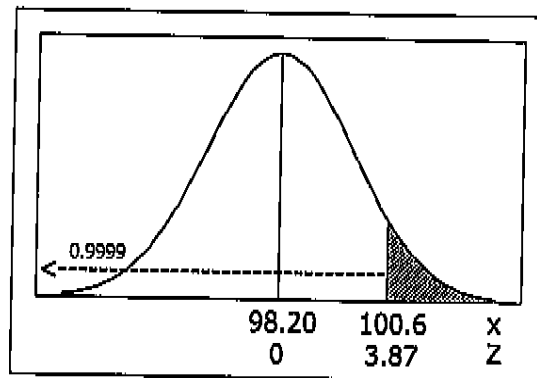


29. normal distribution: $\mu = 98.20$ and $\sigma = 0.62$

a. $P(x > 100.6)$
 $= P(z > 3.87)$
 $= 1 - 0.9999$
 $= 0.0001$

Excel: `1 - NORMDIST(100.6,98.29,0.62,1)`
 $= 5.4E-05 = 0.000054$

Yes. The cut-off is appropriate in that there is a small probability of saying that a healthy person has a fever, but many with low grade fevers may erroneously be labeled healthy.

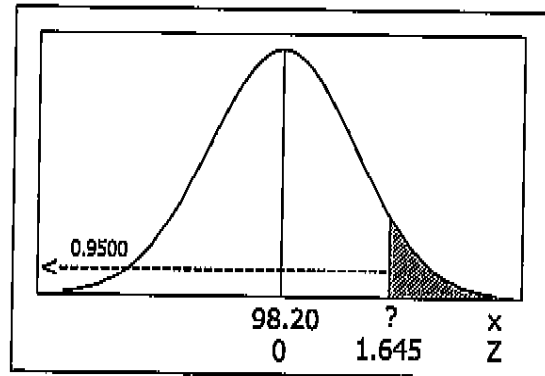


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- b. For the highest 5%, $A = 0.9500$
and $z = 1.645$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 98.20 + (1.645)(0.62) \\ &= 98.20 + 1.02 \\ &= 99.22 \text{ }^\circ\text{F} \end{aligned}$$

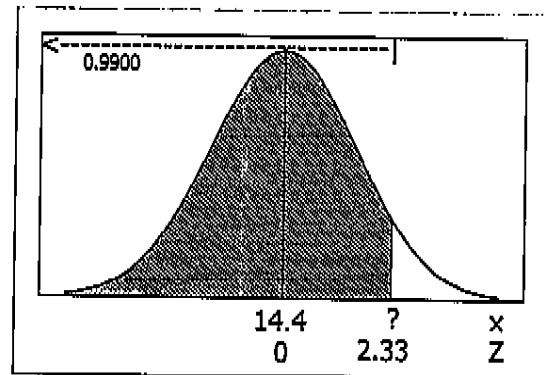
Excel: $\text{NORMINV}(0.95,98.20,0.62) = 99.22$



- ~~30.~~ normal distribution: $\mu = 14.4$ and $\sigma = 1.0$
For P_{99} , $A = 0.9900$ [0.9901]
and $z = 2.33$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 14.4 + (2.33)(1.0) \\ &= 14.4 + 2.3 \\ &= 16.7 \text{ inches} \end{aligned}$$

Excel: $\text{NORMINV}(0.99,14.4,1.0) = 16.7$

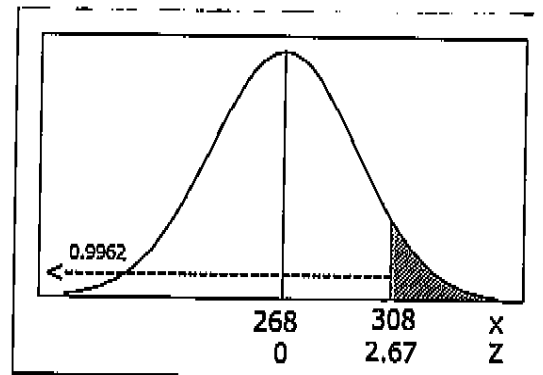


- ~~31.~~ normal distribution: $\mu = 268$ and $\sigma = 15$
a. $P(x > 308)$

$$\begin{aligned} &= P(z > 2.67) \\ &= 1 - 0.9962 \\ &= 0.0038 \end{aligned}$$

Excel: $1 - \text{NORMDIST}(308,268,15,1) = 0.0038$

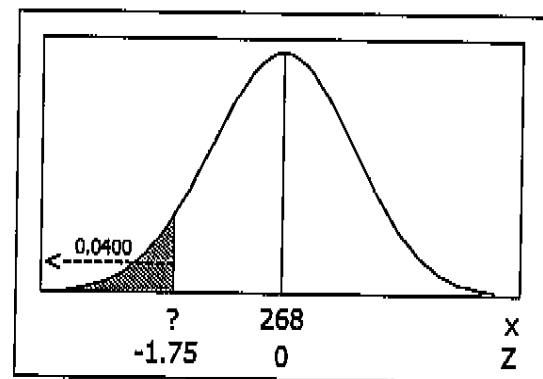
The result suggests that an unusual event has occurred – but certainly not an impossible one, as about 38 of every 10,000 pregnancies can be expected to last as long.



- b. For the lowest 4%, $A = 0.0400$ [0.0401]
and $z = -1.75$.

$$\begin{aligned} x &= \mu + z\sigma \\ &= 268 + (-1.75)(15) \\ &= 268 - 26 \\ &= 242 \text{ days} \end{aligned}$$

Excel: $\text{NORMINV}(0.04,268,15) = 242$



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GROUP 2
SOLUTIONS

6-4 Sampling Distributions and Estimators

1. Given a population distribution of scores, one can take a sample of size n and calculate any of several statistics. A sampling distribution is the distribution of all possible values of such a particular statistic.
2. No. When dealing with one sample of size n , a collection of individual scores, a person is investigating the distribution of the individual scores. In order to investigate the distribution of the means, a person needs a collection of means – which are typically obtained from multiple samples of size n .
3. An unbiased estimator is one whose expected value is the true value of the parameter which it estimates. The sample mean is an unbiased estimator of the population mean because the expected value (or mean) of its sampling distribution is the population mean.
4. In most situations the population is large enough so that repeated sampling will not alter the selection probabilities – and this is equivalent to sampling with replacement. Assuming such is the case allows the use of the simpler mathematical formulas involving independent events instead of the more complicated formulas involving dependent events.
5. No. The students at New York University are not necessarily representative (by race, major, etc.) of the population of all U.S. college students.
6. a. Yes. The sample mean is an unbiased estimator of the population mean.
b. No. The sample median is not an unbiased estimator of the population median.
c. Yes. The sample proportion is an unbiased estimator of the population proportion.
d. Yes. The sample variance is an unbiased estimator of the population variance.
e. No. The sample standard deviation is not an unbiased estimator of the population standard deviation.
f. No. The sample range is not an unbiased estimator of the population range.
7. The sample means will have a distribution that is approximately normal. They will tend to form a symmetric, unimodal and bell-shaped distribution around the value of the population mean.
8. The sample proportions will have a distribution that is approximately normal. They will tend to form a symmetric, unimodal and bell-shaped distribution around the value of the population proportion.

9, 10, 11 & 12

9. a. The medians of the 9 samples are given in column 2 at the right. The sampling distribution of the median is given in columns 3 and 4 at the right.
- b. The population median is 3. The mean of the sample medians is $\sum \bar{x} \cdot P(\bar{x}) = 45/9 = 5$.
- c. In general the sample medians do not target the value of the population median. For this reason, sample median is not a good estimator of the population median.

sample	\bar{x}	\bar{x}	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$
2,2	2	2	1/9	2/9
2,3	2.5	2.5	2/9	5/9
2,10	6	3	1/9	3/9
3,2	2.5	6	2/9	12/9
3,3	3	6.5	2/9	13/9
3,10	6.5	10	1/9	10/9
10,2	6	9/9	45/9	
10,3	6.5			
10,10	10			

Sampling Distributions and Estimators SECTION 6-4 167

NOTE: Section 5-2 defined the mean of a probability distribution of x 's as $\mu_x = \Sigma[x \cdot P(x)]$. If the variable is designated by the symbol y , then the mean of a probability distribution of y 's is $\mu_y = \Sigma[y \cdot P(y)]$. In this section, the variables are statistics – like \bar{X} and \hat{p} . In such cases, the formula for the mean may be adjusted – to $\mu_{\bar{x}} = \Sigma[\bar{x} \cdot P(\bar{x})]$ and $\mu_{\hat{p}} = \Sigma[\hat{p} \cdot P(\hat{p})]$. In a similar manner, the formula for the variance of a probability distribution may also be adjusted to match the variable being considered.



10. a. The standard deviations of the 9 samples are given in column 3 at the right. The sampling distribution of the standard deviation is given in columns 4 and 5 at the right.

sample	s^2	s	s	$P(s)$	$s \cdot P(s)$
2,2	0	0	0	3/9	0.000
2,3	0.5	0.707	0.707	2/9	0.157
2,10	32	5.657	4.950	2/9	1.100
3,2	0.5	0.707	5.657	2/9	1.257
3,3	0	0		9/9	2.514
3,10	24.5	4.950			
10,2	32	5.657			
10,3	24.5	4.950			
10,10	0	0			

b. Since the values 2,3,10 are considered a population, the population variance is $\sigma^2 = \Sigma(x-\mu)^2/N = (3^2 + 2^2 + 5^2)/3 = 38/3$. The population standard deviation is

$\sqrt{38/3} = 3.559$. The mean of the sample standard deviations is $\Sigma s \cdot P(s) = 2.514$.

c. In general the sample standard deviations do not target the value of the population standard deviation. For this reason, the sample standard deviation is not a good estimator of the population standard deviation.



11. a. The variances of the 9 samples are given in column 2 at the right. The sampling distribution of the variance is given in columns 3 and 4 at the right.

sample	s^2	s^2	$P(s^2)$	$s^2 \cdot P(s^2)$
2,2	0	0	3/9	0/9
2,3	0.5	0.5	2/9	1/9
2,10	32	24.5	2/9	49/9
3,2	0.5	32	2/9	64/9
3,3	0		9/9	114/9
3,10	24.5			
10,2	32			
10,3	24.5			
10,10	0			

b. Since the values 2,3,10 are considered a population, the population variance is $\sigma^2 = \Sigma(x-\mu)^2/N = (3^2 + 2^2 + 5^2)/3 = 38/3$. The mean of the sample variances is $\Sigma s^2 \cdot P(s^2) = 114/9 = 38/3$.

c. The sample variance always targets the value of the population variance. For this reason, the sample variance is a good estimator of the population variance.



12. a. The means of the 9 samples are given in column 2 at the right. The sampling distribution of the mean is given in columns 3 and 4 at the right.

sample	\bar{x}	\bar{x}	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$
2,2	2	2	1/9	2/9
2,3	2.5	2.5	2/9	5/9
2,10	6	3	1/9	3/9
3,2	2.5	6	2/9	12/9
3,3	3	6.5	2/9	13/9
3,10	6.5	10	1/9	10/9
10,2	6		9/9	45/9
10,3	6.5			
10,10	10			

b. The population mean is 5. The mean of the sample means is $\Sigma \bar{x} \cdot P(\bar{x}) = 45/9 = 5$.

c. The sample mean always targets the value of the population mean. For this reason, the sample mean is a good estimator of the population mean.

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The information below and the box at the right apply to Exercises 13-16.

original population of scores in order: 46 49 56 58

summary statistics: $N=4$ $\Sigma x = 209$ $\Sigma x^2 = 11017$

$\mu = \Sigma x/N$

$= 209/4 = 52.25$

population $\bar{x} = (x_2+x_3)/2$

$= (49+56)/2 = 52.5$

population $R = x_n - x_1$

$= 58 - 46 = 12$

$\sigma^2 = [\Sigma(x-\mu)^2]/N$

$= [(-6.25)^2 + (-3.25)^2 + (3.75)^2 + (5.75)^2]/4$

$= [39.0625 + 10.5625 + 14.0625 + 33.0625]/4$

$= 96.75/4 = 24.1875$

sample	\bar{x}	\tilde{x}	R	s^2
46,46	46.0	46.0	0	0.0
46,49	47.5	47.5	3	4.5
46,56	51.0	51.0	10	50.0
46,58	52.0	52.0	12	72.0
49,46	47.5	47.5	3	4.5
49,49	49.0	49.0	0	0.0
49,56	52.5	52.5	7	24.5
49,58	53.5	53.5	9	40.5
56,46	51.0	51.0	10	50.0
56,49	52.5	52.5	7	24.5
56,56	56.0	56.0	0	0.0
56,58	57.0	57.0	2	2.0
58,46	52.0	52.0	12	72.0
58,49	53.5	53.5	9	40.5
58,56	57.0	57.0	2	2.0
58,58	58.0	58.0	0	0.0

13. a. The sixteen possible samples are given in the "samples" column of the box preceding this exercise
 b. The sixteen possible means are given in column 2 of the box preceding this exercise. The sampling distribution of the mean is given in the first two columns at the right.
 c. The population mean is 52.25. The mean of the sample means is $\Sigma \bar{x} \cdot P(\bar{x}) = 836.0/16 = 52.25$. They are the same.
 d. Yes. The sample mean always targets the value of the population mean. For this reason, the sample mean is a good estimator of the population mean.

\bar{x}	$P(\bar{x})$	$\bar{x} \cdot P(\bar{x})$
46.0	1/16	46.0/16
47.5	2/16	95.0/16
49.0	1/16	49.0/16
51.0	2/16	102.0/16
52.0	2/16	104.0/16
52.5	2/16	105.0/16
53.5	2/16	107.0/16
56.0	1/16	56.0/16
57.0	2/16	114.0/16
58.0	1/16	58.0/16
	16/16	836.0/16

14. a. The sixteen possible samples are given in the "samples" column of the box preceding Exercise 13.
 b. The sixteen possible medians are given in column 3 of the box preceding Exercise 13. The sampling distribution of the median is given in the first two columns at the right.
 c. The population median is 52.5. The mean of the sample medians is $\Sigma \tilde{x} \cdot P(\tilde{x}) = 836.0/16 = 52.25$. They are not the same.
 d. No. The sample medians do not always target the value of the population median. For this reason, the sample median is not a good estimator of the population median.

\tilde{x}	$P(\tilde{x})$	$\tilde{x} \cdot P(\tilde{x})$
46.0	1/16	46.0/16
47.5	2/16	95.0/16
49.0	1/16	49.0/16
51.0	2/16	102.0/16
52.0	2/16	104.0/16
52.5	2/16	105.0/16
53.5	2/16	107.0/16
56.0	1/16	56.0/16
57.0	2/16	114.0/16
58.0	1/16	58.0/16
	16/16	836.0/16

15. a. The sixteen possible samples are given in the "samples" column of the box preceding Exercise 13.

Group 3 Solutions 9 and 11

The Central Limit Theorem SECTION 6-5 173

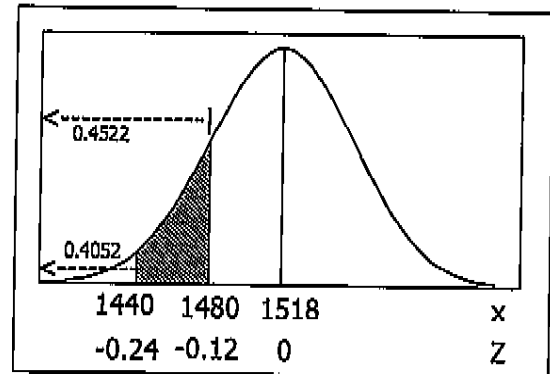
8. a. normal distribution

$$\mu = 1518$$

$$\sigma = 325$$

$$\begin{aligned} P(1440 < x < 1480) \\ &= P(-0.24 < z < -0.12) \\ &= 0.4522 - 0.4052 \\ &= 0.0470 \end{aligned}$$

$$\begin{aligned} \text{NORMDIST}(1480, 1518, 325, 1) - \\ \text{NORMDIST}(1440, 1518, 325, 1) = 0.0483 \end{aligned}$$



b. normal distribution,

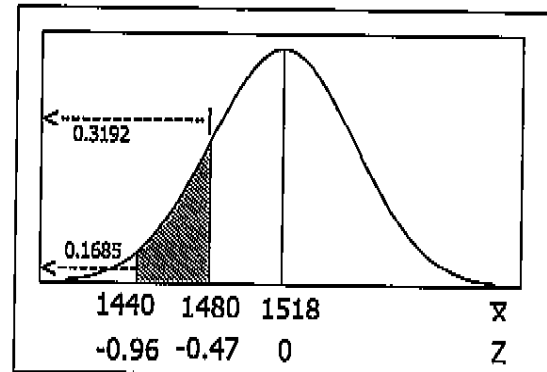
since the original distribution is so

$$\mu_{\bar{x}} = \mu = 1518$$

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 325 / \sqrt{16} = 81.25$$

$$\begin{aligned} P(1440 < \bar{x} < 1480) \\ &= P(-0.96 < z < -0.47) \\ &= 0.3192 - 0.1685 \\ &= 0.1507 \end{aligned}$$

$$\begin{aligned} \text{NORMDIST}(1480, 1518, 325/\text{SQRT}(16), 1) - \\ \text{NORMDIST}(1440, 1518, 325/\text{SQRT}(16), 1) \\ = 0.1515 \end{aligned}$$



c. Since the original distribution is normal, the Central Limit Theorem can be used in part (b) even though the sample size does not exceed 30.



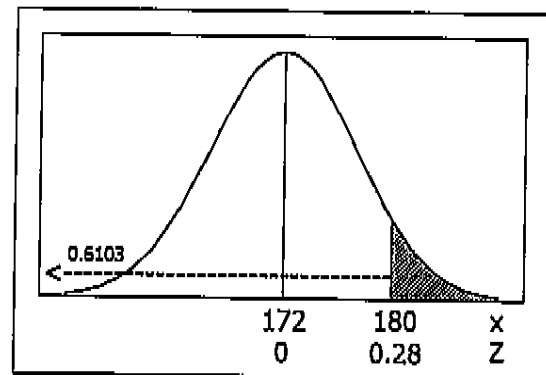
9. a. normal distribution

$$\mu = 172$$

$$\sigma = 29$$

$$\begin{aligned} P(x > 180) \\ &= P(z > 0.28) \\ &= 1 - 0.6103 \\ &= 0.3897 \end{aligned}$$

$$1 - \text{NORMDIST}(180, 172, 29, 1) = 0.3913$$



b. normal distribution,

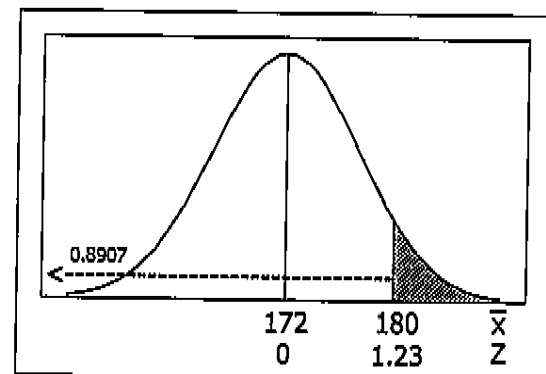
since the original distribution is so

$$\mu_{\bar{x}} = \mu = 172$$

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 29 / \sqrt{20} = 6.48$$

$$\begin{aligned} P(\bar{x} > 180) \\ &= P(z > 1.23) \\ &= 1 - 0.8907 \\ &= 0.1093 \end{aligned}$$

$$\begin{aligned} 1 - \text{NORMDIST}(180, 172, 29/\text{SQRT}(20), 1) \\ = 0.1087 \end{aligned}$$



c. Yes. A capacity of 20 is not appropriate when the passengers are all adult men, since a 10.93% probability of overloading is too much of a risk.

174 CHAPTER 6 Normal Probability Distributions

10. a. normal distribution

$$\mu = 100$$

$$\sigma = 15$$

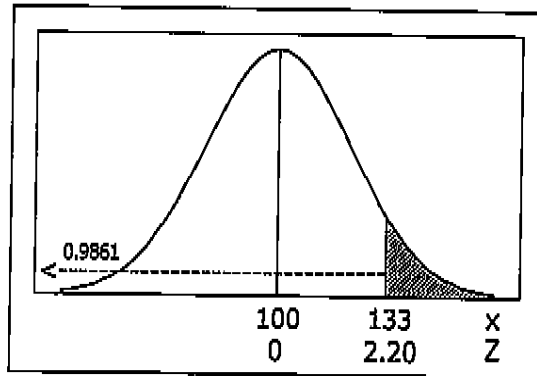
$$P(x > 133)$$

$$= P(z > 2.20)$$

$$= 1 - 0.9861$$

$$= 0.0139$$

$$1 - \text{NORMDIST}(133, 100, 15, 1) = 0.0139$$



b. normal distribution,

since the original distribution is so

$$\mu_{\bar{x}} = \mu = 100$$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 15/\sqrt{9} = 5$$

$$P(\bar{x} > 133)$$

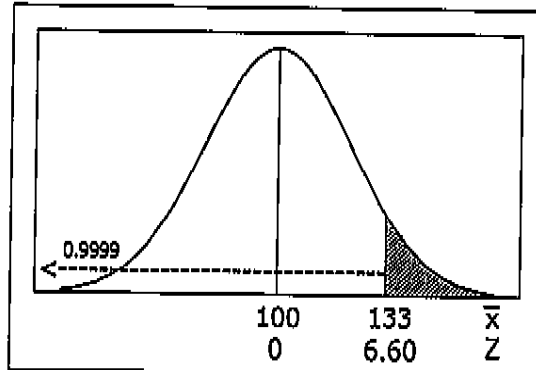
$$= P(z > 6.60)$$

$$= 1 - 0.9999$$

$$= 0.0001$$

$$1 - \text{NORMDIST}(133, 100, 15/\text{SQRT}(9), 1)$$

$$= 2.1\text{E-}11$$



c. No. Even though the mean score is 133, some of the individual scores may be below 131.5.



11. a. normal distribution

$$\mu = 172$$

$$\sigma = 29$$

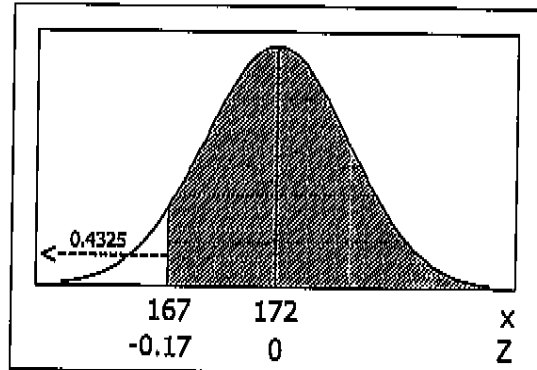
$$P(x > 167)$$

$$= P(z > -0.17)$$

$$= 1 - 0.4325$$

$$= 0.5675$$

$$1 - \text{NORMDIST}(167, 172, 29, 1) = 0.5684$$



b. normal distribution,

since the original distribution is so

$$\mu_{\bar{x}} = \mu = 172$$

$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 29/\sqrt{12} = 8.372$$

$$P(\bar{x} > 167)$$

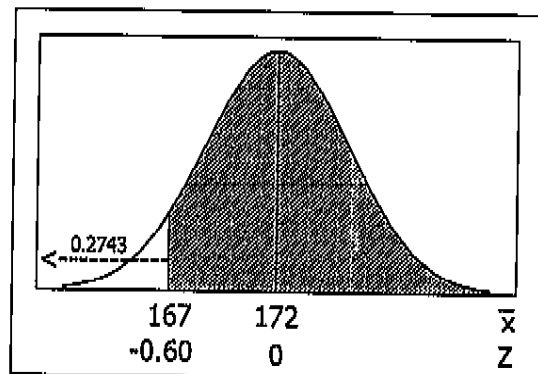
$$= P(z > -0.60)$$

$$= 1 - 0.2743$$

$$= 0.7257$$

$$1 - \text{NORMDIST}(167, 172, 29/\text{SQRT}(12), 1)$$

$$= 0.7248$$



c. No. It appears that the 12 person capacity

could easily exceed the 2004 lbs -

especially when the weight of clothes and

equipment is considered. On the other hand, skiers may be lighter than the general

population - as the skiing may not be an activity that attracts heavier persons.

Group 4 solutions

184 CHAPTER 6 Normal Probability Distributions

21 and 27

18. binomial: $n=2822$ and $p=0.75$

normal approximation appropriate since

$$np = 2822(0.75) = 2116.5 \geq 5$$

$$nq = 2822(0.25) = 705.5 \geq 5$$

$$\mu = np = 2822(0.75) = 2116.5$$

$$\sigma = \sqrt{npq} = \sqrt{2822(0.75)(0.25)} = 23.003$$

$$P(x \leq 2060)$$

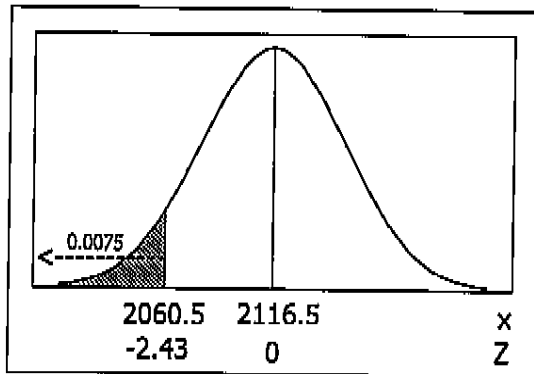
$$= P(x < 2060.5)$$

$$= P(z < -2.43) = 0.0075$$

$$\text{BINOMDIST}(2060, 2822, 0.75, 1) = 0.0078$$

Yes. Since $0.0075 \leq 0.05$, it would be

unusual to get 2060 Internet users or less if the true population proportion were really 0.75.

19. binomial: $n=574$ and $p=0.50$

normal approximation appropriate since

$$np = 574(0.50) = 287 \geq 5$$

$$nq = 574(0.50) = 287 \geq 5$$

$$\mu = np = 574(0.50) = 287$$

$$\sigma = \sqrt{npq} = \sqrt{574(0.50)(0.50)} = 11.979$$

$$P(x \geq 525)$$

$$= P(x > 524.5)$$

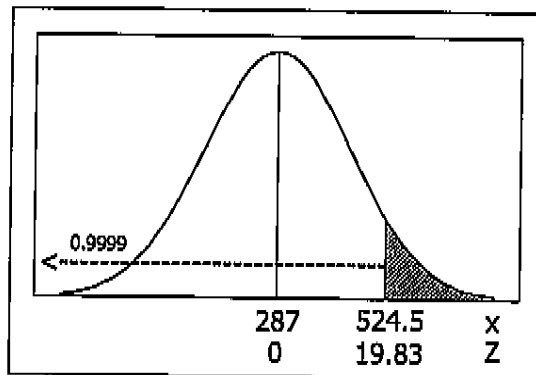
$$= P(z > 19.83)$$

$$= 1 - 0.9999$$

$$= 0.0001$$

$$1 - \text{BINOMDIST}(524, 574, 0.50, 1) = 1.4E-14$$

Yes. Since the probability of getting at least 525 girls by chance alone is so small, it appears that the method is effective and that the genders were not being determined by chance alone.

20. binomial: $n=152$ and $p=0.50$

normal approximation appropriate since

$$np = 152(0.50) = 76 \geq 5$$

$$nq = 152(0.50) = 76 \geq 5$$

$$\mu = np = 152(0.50) = 76$$

$$\sigma = \sqrt{npq} = \sqrt{152(0.50)(0.50)} = 6.164$$

$$P(x \geq 127)$$

$$= P(x > 126.5)$$

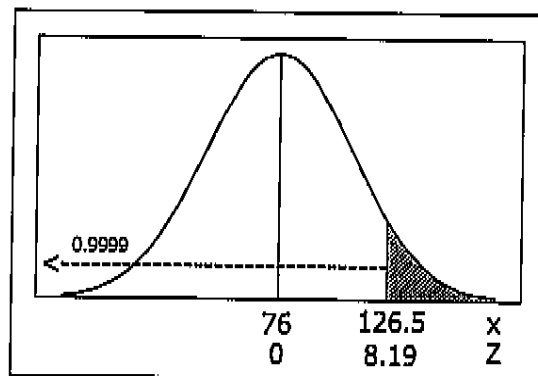
$$= P(z > 8.19)$$

$$= 1 - 0.9999$$

$$= 0.0001$$

$$1 - \text{BINOMDIST}(126, 152, 0.50) = 3.8E-15$$

Yes. Since the probability of getting at least 127 boys by chance alone is so small, it appears that the method is effective and that the genders were not being determined by chance alone.



*

21. binomial: $n=580$ and $p=0.25$

normal approximation appropriate since

$$np = 580(0.25) = 145 \geq 5$$

$$nq = 580(0.75) = 435 \geq 5$$

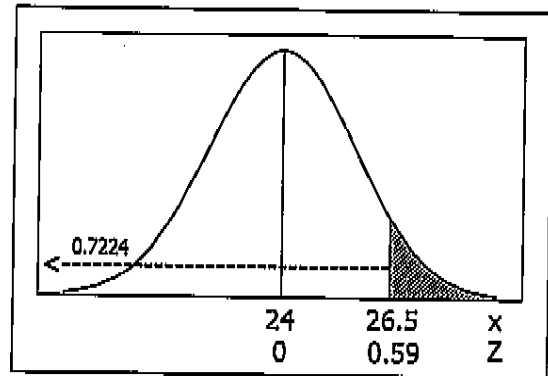
$$\mu = np = 580(0.25) = 145$$

$$\sigma = \sqrt{npq} = \sqrt{580(0.25)(0.75)} = 10.428$$

Normal as Approximation to Binomial SECTION 6-6 187



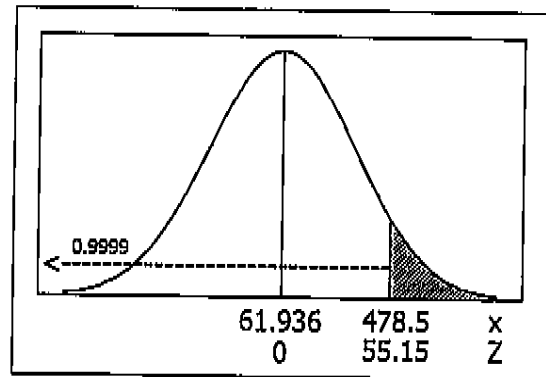
27. binomial: $n=100$ and $p=0.24$
 normal approximation appropriate since
 $np = 100(0.24) = 24 \geq 5$
 $nq = 100(0.76) = 76 \geq 5$
 $\mu = np = 100(0.24) = 24$
 $\sigma = \sqrt{npq} = \sqrt{100(0.24)(0.76)} = 4.271$
 $P(x \geq 27)$
 $= P(x > 26.5)$
 $= P(z > 0.59)$
 $= 1 - 0.7224$
 $= 0.2776$



$$1 - \text{BINOMDIST}(26, 100, 0.24, 1) = 0.2748$$

No. Since $0.2776 > 0.05$, 27 is not an unusually high number of blue M&M's.

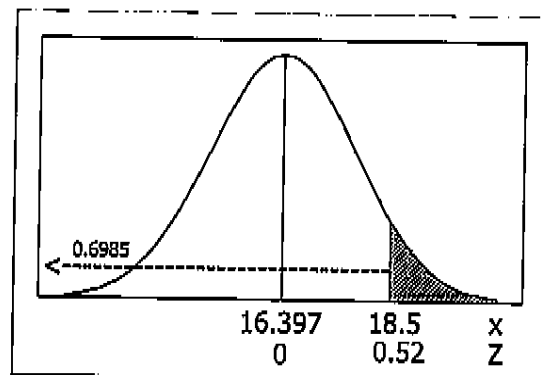
28. binomial: $n=784$ and $p=0.079$
 normal approximation appropriate since
 $np = 784(0.079) = 61.936 \geq 5$
 $nq = 784(0.921) = 722.064 \geq 5$
 $\mu = np = 784(0.079) = 61.936$
 $\sigma = \sqrt{npq} = \sqrt{784(0.079)(0.921)} = 7.553$
 $P(x \geq 479)$
 $= P(x > 478.5)$
 $= P(z > 55.15)$
 $= 1 - 0.9999$
 $= 0.0001$



$$1 - \text{BINOMDIST}(478, 784, 0.079, 1) = -2.4E-14 \text{ [NOTE: THIS IS A RARE ROUND-OFF ERROR IN EXCEL. PROBABILITY CANNOT BE NEGATIVE. THE ANSWER IS ESSENTIALLY 0]}$$

Yes. Since the probability of obtaining 479 or more such checks from normal honest transactions by chance alone is so very strong, there is strong evidence to indicate that the checks from the suspected companies do not follow the normal pattern and are likely fraudulent.

29. binomial: $n=863$ and $p=0.019$
 normal approximation appropriate since
 $np = 863(0.019) = 16.397 \geq 5$
 $nq = 863(0.981) = 846.603 \geq 5$
 $\mu = np = 863(0.019) = 16.397$
 $\sigma = \sqrt{npq} = \sqrt{863(0.019)(0.981)} = 4.011$
 $P(x \geq 19)$
 $= P(x > 18.5)$
 $= P(z > 0.52)$
 $= 1 - 0.6985$
 $= 0.3015$



$$1 - \text{BINOMDIST}(18, 863, 0.19, 1) = 0.2900$$

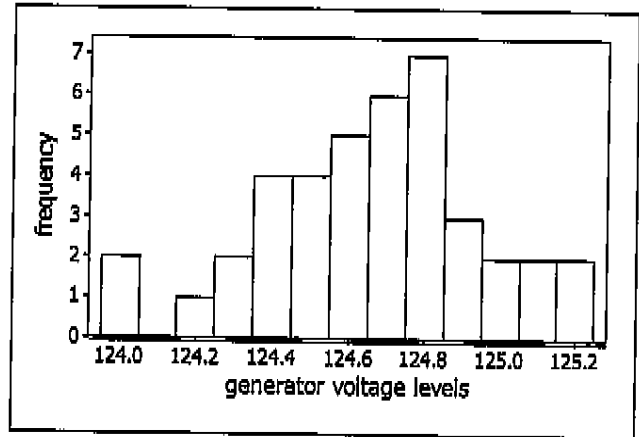
Since the $0.3015 > 0.05$, 19 or more persons experiencing flu symptoms is not an unusual occurrence for a normal population. There is no evidence to suggest that flu symptoms are an adverse reaction to the drug.

Group 5 solution

13 only

12. Yes, the data appear to come from a population with a normal distribution. The frequency distribution and the histogram indicate the data is approximately bell-shaped.

volts	frequency
124.0	2
124.1	0
124.2	1
124.3	2
124.4	4
124.5	4
124.6	5
124.7	6
124.8	7
124.9	3
125.0	2
125.1	2
125.2	2
	<u>40</u>



NOTE: The "normal quantile plots" given on the left for exercises 13-16 may be obtained directly from many sources or constructed using Minitab for n scores in C1 using the commands at the right, where (2n-1) and (2n) are the actual values, and then plotting C1 on the x-axis and C4 on the y-axis. The "normal probability plots" produced by Excel, as described in the text, are also given on the right for exercises 13-16. See Exercises 19 and 20 for more detail on the mechanics of the normal quantile plot process, which is the five-step "manual construction" process given in the text.

```
MTB> Sort C1 C1
MTB> Set C2
DATA> 1:(2n-1)/2
DATA> end
MTB> Let C3 = C2/(2n)
MTB> INVCDF C3 C4
```



13. Yes. Since the points approximate a straight line, the data appear to come from a population with a normal distribution. The gaps/groupings in the durations may reflect the fact that the times have to reflect whole numbers of orbits or other physical constraints.

