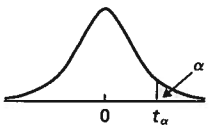
TABLE IV
Values of t_α



df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.309	1.696	2.040	2.453	2.744	31
32	1.309	1.694	2.037	2.449	2.738	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.434	2.719	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.704	40
41	1.303	1.683	2.020	2.421	2.701	41
42	1.302	1.682	2.018	2.418	2.698	42
43	1.302	1.681	2.017	2.416	2.695	43
44	1.301	1.680	2.015	2.414	2.692	44
45	1.301	1.679	2.014	2.412	2.690	45
46	1.300	1.679	2.013	2.410	2.687	46
47	1.300	1.678	2.012	2.408	2.685	47
48	1.299	1.677	2.011	2.407	2.682	48
49	1.299	1.677	2.010	2.405	2.680	49

TABLE IV (cont.)
Values of t_α

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
50	1.299	1.676	2.009	2.403	2.678	*50*
51	1.298	1.675	2.008	2.402	2.676	*51*
52	1.298	1.675	2.007	2.400	2.674	*52*
53	1.298	1.674	2.006	2.399	2.672	*53*
54	1.297	1.674	2.005	2.397	2.670	*54*
55	1.297	1.673	2.004	2.396	2.668	*55*
56	1.297	1.673	2.003	2.395	2.667	*56*
57	1.297	1.672	2.002	2.394	2.665	*57*
58	1.296	1.672	2.002	2.392	2.663	*58*
59	1.296	1.671	2.001	2.391	2.662	*59*
60	1.296	1.671	2.000	2.390	2.660	*60*
61	1.296	1.670	2.000	2.389	2.659	*61*
62	1.295	1.670	1.999	2.388	2.657	*62*
63	1.295	1.669	1.998	2.387	2.656	*63*
64	1.295	1.669	1.998	2.386	2.655	*64*
65	1.295	1.669	1.997	2.385	2.654	*65*
66	1.295	1.668	1.997	2.384	2.652	*66*
67	1.294	1.668	1.996	2.383	2.651	*67*
68	1.294	1.668	1.995	2.382	2.650	*68*
69	1.294	1.667	1.995	2.382	2.649	*69*
70	1.294	1.667	1.994	2.381	2.648	*70*
71	1.294	1.667	1.994	2.380	2.647	*71*
72	1.293	1.666	1.993	2.379	2.646	*72*
73	1.293	1.666	1.993	2.379	2.645	*73*
74	1.293	1.666	1.993	2.378	2.644	*74*
75	1.293	1.665	1.992	2.377	2.643	*75*
80	1.292	1.664	1.990	2.374	2.639	*80*
85	1.292	1.663	1.988	2.371	2.635	*85*
90	1.291	1.662	1.987	2.368	2.632	*90*
95	1.291	1.661	1.985	2.366	2.629	*95*
100	1.290	1.660	1.984	2.364	2.626	*100*
200	1.286	1.653	1.972	2.345	2.601	*200*
300	1.284	1.650	1.968	2.339	2.592	*300*
400	1.284	1.649	1.966	2.336	2.588	*400*
500	1.283	1.648	1.965	2.334	2.586	*500*
600	1.283	1.647	1.964	2.333	2.584	*600*
700	1.283	1.647	1.963	2.332	2.583	*700*
800	1.283	1.647	1.963	2.331	2.582	*800*
900	1.282	1.647	1.963	2.330	2.581	*900*
1000	1.282	1.646	1.962	2.330	2.581	*1000*
2000	1.282	1.646	1.961	2.328	2.578	*2000*
	1.282	1.645	1.960	2.326	2.576	
	$z_{0.10}$	$z_{0.05}$	$z_{0.025}$	$z_{0.01}$	$z_{0.005}$	

Exercise 13.5
on page 651

Solution To find this χ^2-value, we use Table VII. The number of degrees of freedom is 12, so we first go down the outside columns, labeled df, to "12." Then, going across that row to the column labeled $\chi^2_{0.025}$, we reach 23.337. This number is the χ^2-value having area 0.025 to its right, as shown in Fig. 13.2(b). In other words, for a χ^2-curve with df = 12, $\chi^2_{0.025} = 23.337$.

• • •

Exercises 13.1

Understanding the Concepts and Skills

13.1 What is meant by saying that a variable has a chi-square distribution?

13.2 How do you identify different chi-square distributions?

13.3 Consider two χ^2-curves with degrees of freedom 12 and 20, respectively. Which one more closely resembles a normal curve? Explain your answer.

13.4 The t-table has entries for areas of 0.10, 0.05, 0.025, 0.01, and 0.005. In contrast, the χ^2-table has entries for those areas and for 0.995, 0.99, 0.975, 0.95, and 0.90. Explain why the t-values corresponding to these additional areas can be obtained from the existing t-table, but must be provided explicitly in the χ^2-table.

In Exercises 13.5–13.8, use Table VII to determine the required χ^2-values. Illustrate your work graphically.

13.5 For a χ^2-curve with 19 degrees of freedom, find the χ^2-value that has area

a. 0.025 to its right. **b.** 0.95 to its right.

13.6 For a χ^2-curve with 22 degrees of freedom, find the χ^2-value that has area

a. 0.01 to its right. **b.** 0.995 to its right.

13.7 For a χ^2-curve with df = 10, determine

a. $\chi^2_{0.05}$. **b.** $\chi^2_{0.975}$.

13.8 For a χ^2-curve with df = 4, determine

a. $\chi^2_{0.005}$. **b.** $\chi^2_{0.99}$.

Extending the Concepts and Skills

13.9 Explain how you would use Table VII to find the χ^2-value that has area 0.05 to its left. Obtain this χ^2-value for a χ^2-curve with df = 26.

13.10 Explain how you would use Table VII to find the two χ^2-values that divide the area under a χ^2-curve into a middle 0.95 area and two outside 0.025 areas. Find these two χ^2-values for a χ^2-curve with df = 14.

13.2 Chi-Square Goodness-of-Fit Test

Our first chi-square procedure is called the **chi-square goodness-of-fit test.** We can use this procedure to perform a hypothesis test about the distribution of a qualitative (categorical) variable or a discrete quantitative variable that has only finitely many possible values.[†] We introduce and explain the reasoning behind the chi-square goodness-of-fit test next.

Example 13.2 | Introduces the Chi-Square Goodness-of-Fit Test

Violent Crimes The U.S. Federal Bureau of Investigation (FBI) compiles data on crimes and crime rates and publishes the information in *Crime in the United States*. A violent crime is classified by the FBI as murder, forcible rape, robbery,

[†]Actually, the chi-square goodness-of-fit test can be applied to any variable whose possible values have been grouped into a finite number of categories.

INSTRUCTIONS 13.1 Steps for generating Output 13.1

MINITAB

1 Store the violent-crime types, relative frequencies, and observed frequencies in columns named CRIME, P, and O, respectively
2 Choose **Stat ➤ Tables ➤ Chi-Square Goodness-of-Fit Test...**
3 Select the **Observed counts** option button
4 Specify O in the **Observed counts** text box
5 Specify CRIME in the **Category names** text box
6 Select the **Specific proportions** option button from the **Test** list
7 Specify P in the **Specific proportions** text box
8 Click **OK**

EXCEL

1 Store the violent-crime types, relative frequencies, and observed frequencies in ranges named CRIME, P, and O, respectively
2 Choose **DDXL ➤ Tables**
3 Select **Goodness of Fit** from the **Function type** drop-down list box
4 Specify CRIME in the **Category Names** text box
5 Specify O in the **Observed Counts** text box
6 Specify P in the **Test Distribution** text box
7 Click **OK**

TI-83/84 PLUS

1 Store the observed frequencies and relative frequencies in Lists 1 and 2, respectively
2 Press **PRGM**
3 Arrow down to CHIGFT and press **ENTER** twice

■ ■ ■

Exercises 13.2

Understanding the Concepts and Skills

13.11 Why is the phrase "goodness of fit" used to describe the type of hypothesis test considered in this section?

13.12 Are the observed frequencies variables? What about the expected frequencies? Explain your answers.

In each of Exercises 13.13–13.18, we have given the relative frequencies for the null hypothesis of a chi-square goodness-of-fit test and the sample size. In each case, decide whether Assumptions 1 and 2 for using that test are satisfied.

13.13 Sample size: $n = 100$.
Relative frequencies: 0.65, 0.30, 0.05.

13.14 Sample size: $n = 50$.
Relative frequencies: 0.65, 0.30, 0.05.

13.15 Sample size: $n = 50$.
Relative frequencies: 0.20, 0.20, 0.25, 0.30, 0.05.

13.16 Sample size: $n = 50$.
Relative frequencies: 0.22, 0.21, 0.25, 0.30, 0.02.

13.17 Sample size: $n = 50$.
Relative frequencies: 0.22, 0.22, 0.25, 0.30, 0.01.

13.18 Sample size: $n = 100$.
Relative frequencies: 0.44, 0.25, 0.30, 0.01.

13.19 Primary Heating Fuel. According to *Current Housing Reports*, published by the U.S. Census Bureau, the primary heating fuel for all occupied housing units is distributed as follows.

Primary heating fuel	Percentage
Utility gas	51.5
Fuel oil, kerosene	9.8
Electricity	30.7
Bottled, tank, or LPG	5.7
Wood and other fuel	1.9
None	0.4

Suppose that you want to determine whether the distribution of primary heating fuel for occupied housing units built after 2000 differs from that of all occupied housing units. To decide, you take a random sample of housing units built after 2000 and obtain a frequency distribution of their primary heating fuel.

a. Identify the population and variable under consideration here.
b. For each of the following sample sizes, determine whether conducting a chi-square goodness-of-fit test is appropriate and explain your answers: 200; 250; 300.
c. Strictly speaking, what is the smallest sample size for which conducting a chi-square goodness-of-fit test is appropriate?

In each of Exercises 13.20–13.25, we have provided a distribution and the observed frequencies of the values of a variable from a simple random sample of a population. In each case, use the chi-square goodness-of-fit test to decide, at the specified significance level, whether the distribution of the variable differs from the given distribution.

13.20 Distribution: 0.2, 0.4, 0.3, 0.1;
Observed frequencies: 39, 78, 64, 19;
Significance level = 0.05

13.21 Distribution: 0.2, 0.4, 0.3, 0.1;
Observed frequencies: 85, 215, 130, 70;
Significance level = 0.05

13.22 Distribution: 0.2, 0.1, 0.1, 0.3, 0.3;
Observed frequencies: 29, 13, 5, 25, 28;
Significance level = 0.10

13.23 Distribution: 0.2, 0.1, 0.1, 0.3, 0.3;
Observed frequencies: 9, 7, 1, 12, 21;
Significance level = 0.10

13.24 Distribution: 0.5, 0.3, 0.2;
Observed frequencies: 45, 39, 16;
Significance level = 0.01

13.25 Distribution: 0.5, 0.3, 0.2;
Observed frequencies: 147, 115, 88;
Significance level = 0.01

In each of Exercises 13.26–13.31, apply the chi-square goodness-of-fit test, using either the critical-value approach or the P-value approach, to perform the required hypothesis test.

13.26 Population by Region. According to the U.S. Census Bureau publication *Demographic Profiles*, a relative-frequency distribution of the U.S. resident population by region in 2000 was as follows.

Region	Northeast	Midwest	South	West
Rel. freq.	0.190	0.229	0.356	0.225

A simple random sample of this year's U.S. residents gave the following frequency distribution.

Region	Northeast	Midwest	South	West
Frequency	50	43	111	46

At the 5% significance level, do the data provide sufficient evidence to conclude that this year's resident population distribution by region has changed from the 2000 distribution?

13.27 Freshmen Politics. The Higher Education Research Institute of the University of California, Los Angeles, publishes information on characteristics of incoming college freshmen in *The American Freshman*. In 2000, 27.7% of incoming freshmen characterized their political views as liberal, 51.9% as moderate, and 20.4% as conservative. For this year, a random sample of 500 incoming college freshmen yielded the following frequency distribution for political views.

Political view	Frequency
Liberal	160
Moderate	246
Conservative	94

a. Identify the population and variable under consideration here.
b. At the 5% significance level, do the data provide sufficient evidence to conclude that this year's distribution of political views for incoming college freshmen has changed from the 2000 distribution?
c. Repeat part (b), using a significance level of 10%.

13.28 Car Sales. The American Automobile Manufacturers Association compiles data on U.S. car sales by type of car. Following is the 1990 distribution, as reported in the *World Almanac*.

Type of car	Small	Midsize	Large	Luxury
Percentage	32.8	44.8	9.4	13.0

A random sample of last year's U.S. car sales yielded the following data.

Type of car	Small	Midsize	Large	Luxury
Frequency	133	249	47	71

a. Identify the population and variable under consideration here.
b. At the 5% significance level, do the data provide sufficient evidence to conclude that last year's type-of-car distribution for U.S. car sales differs from the 1990 distribution?

13.29 M&M Colors. Observing that the proportion of blue M&Ms in his bowl of candy appeared to be less than that of the other colors, Ronald D. Fricker, Jr., decided to compare the color distribution in randomly chosen bags of M&Ms to the theoretical distribution reported by M&M/MARS consumer affairs. Fricker published his findings in the article "The Mysterious Case of the Blue M&Ms"