*Exercise 5.2.6.** *Square Well Potential.* Consider a particle in a square well potential:

$$V(x)=\begin{cases}0, & |x|\le a\\ V_0, & |x|\ge a\end{cases}$$

Since when $V_0\to\infty$, we have a box, let us guess what the lowering of the walls does to the states. First of all, all the bound states (which alone we are interested in), will have $E\le V_0$. Second, the wave functions of the low-lying levels will look like those of the particle in a box, with the obvious difference that ψ will not vanish at the walls but instead spill out with an exponential tail. The eigenfunctions will still be even, odd, even, etc.

(1) Show that the even solutions have energies that satisfy the transcendental equation

$$k\tan ka=\kappa \tag{5.2.23}$$

while the odd ones will have energies that satisfy

$$k\cot ka=-\kappa \tag{5.2.24}$$

where k and $i\kappa$ are the real and complex wave numbers inside and outside the well, respectively. Note that k and κ are related by

$$k^2+\kappa^2=2mV_0/\hbar^2 \tag{5.2.25}$$

Verify that as V_0 tends to ∞, we regain the levels in the box.

(2) Equations (5.2.23) and (5.2.24) must be solved graphically. In the ($\alpha=ka$, $\beta=\kappa a$) plane, imagine a circle that obeys Eq. (5.2.25). The bound states are then given by the intersection of the curve $\alpha\tan\alpha=\beta$ or $\alpha\cot\alpha=-\beta$ with the circle. (Remember α and β are positive.)

(3) Show that there is always one even solution and that there is no odd solution unless $V_0\ge\hbar^2\pi^2/8ma^2$. What is E when V_0 just meets this requirement? Note that the general result from Exercise 5.2.2b holds.