Consider a 1-D free particle, describable as a wave packet at initial time t_0.

a) Show, applying Ehrenfest's theorem, that $\langle X \rangle$ is a linear function of time and $\langle P \rangle$ is a constant.

b) Write the equations of motion for the mean values $\langle X^2 \rangle$ and $\langle XP + PX \rangle$. Integrate these equations.

c) Show that, with a suitable choice of the time origin, the rms deviation ΔX is given by

$$(\Delta X)^2 = \frac{1}{m^2}(\Delta P)_0^2 t^2 + (\Delta X)_0^2$$

where $(\Delta X)_0^2$ and $(\Delta P)_0^2$ are the rms deviations at time t_0. How does the width of the wave packet vary as a function of time?

d) Give a physical interpretation.