6. Let A be a non-empty bounded subset of $\mathbb{R}$. Let $B = \{2a^2 : a \in A\}$. Prove that B is bounded above. (9 points)

What do you mean by this?

Try 1

Let $a+1$ be bounded above.

then $2a^2 > a = 2(a+1)^2 = 2(a^2+2a+1)$
$> 2a^2+2a+1 > 2a > a$

by def. of sup

$2a < a$.

$\therefore$ This is a contradiction.

2

Try 2

Let $2a^2 \in B$ and $a \in A$.

If $A, B \in \mathbb{R}$.

$\alpha < A < \beta$ $\qquad$ $\delta < B < \gamma$

Since $2a^2 > a$ for $a \in \mathbb{R}$.

$\delta - \alpha < B - A < \gamma - \beta$.

$\sup B - \inf A$ s.t B is bounded above.