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MATH 320, Fall 2009
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Exam 1

Due in class on October 2, 2009.

1. Consider an electric circuit containing a capacitor, resistor and battery. The charge $Q(t)$ on the capacitor satisfies the equation resulting from Kirchhoff's laws

$$R\frac{dQ}{dt} + \frac{Q}{C} = V(t)$$

where R is the resistance, C is the capacitance and $V(t)$ is the voltage supplied by the battery.

   (a) Find the general solution if $R = 500$ ohms, $C = 10^{-5}$ farad, and $V(t) = 5t$ volts (t is dimensionless here) without using Matlab.

   (b) If the initial charge is 10^{-4} coulomb, find the charge $Q(t)$ as a function of time.

2. Find the value of a for which the equation

$$ae^{2x}\sin y + 2y + (3e^{2x}\cos y + 2x + 3y)y' = 0$$

is exact. Solve the equation using that value of a.

3. A body falling in a relatively dense fluid, oil for example, is acted on by three forces: the weight W due to gravity (acting downwards), a resistive force R, and a buoyant force B (both acting upwards). The weight W of the object of mass m is mg. The buoyant force B is equal to the weight of the fluid displaced by the object. If the mass of the fluid is m_f, this force is $B = m_f g$. For a slowly moving spherical body of radius r, the resistive force is given by Stokes law $R = 6\pi\mu r v$, where v is the velocity of the body and μ is the coefficient of viscosity of the surrounding fluid.

   (a) Write a differential equation for the velocity that the object has at any given time t.

   (b) If the product $6\pi\mu$ is equal to 3.2, the mass of the spherical object of radius 0.05 meters is 0.5 kg and the mass of the fluid is 0.1 kg, solve the differential equation from part a) without using Matlab.

   (c) Find the velocity 5 seconds after the object is dropped with initial velocity of 0.3 meters per second.

   (d) Sketch the graphs of the general solutions (note that the equation is autonomous) and use the graph to estimate the limiting velocity regardless of the initial velocity.

4. Solve the following differential equations without using Matlab.

   (a) $y' = \frac{2y^2+5x^2}{xy}$ (b) $x^2y' + 3xy = 4y^3$. Solve your answers for y.

5. A second order chemical reaction involves interaction of one molecule of a substance P with one molecule of a substance Q to produce one molecule of a new substance X. This is denoted by $P + Q \to X$. Suppose that p and q are initial concentrations of P and Q respectively (in $\mu g/cm^3$) and $x(t)$ is the concentration of X at time t (in seconds). Then $p - x(t)$ and $q - x(t)$ are concentrations of P and Q at time t respectively. The rate at which the reaction occurs is proportional to the product of these two concentrations.