4. We proved the following theorem in the class: "If $a > 0$ and if a sequence $\langle b_n \rangle_{n=1}^{\infty}$ is convergent, then the sequence $\langle a^{b_n} \rangle_{n=1}^{\infty}$ is convergent." In proving this theorem, we proved that $\langle a^{b_n} \rangle_{n=1}^{\infty}$ is Cauchy instead of proving it converges directly. Why did we have to do that?

By definition,
A sequence is said to be Cauchy whenever it's convergent.
Also, A sequence that is bounded & convergent are cauchy.
By def. of subsequence.

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A subsequence $\langle a^{b_n} \rangle$ of Cauchy sequence $\langle b_n \rangle$ is convergent.
Therefore, if we proved the sequence is "cauchy", then its subsequence will also be convergent.

$\langle a_1, a_2, \ldots, a_n \ldots, + a_N \rangle - \langle a_1 + \ldots a_N \rangle$

$|a_n - a_m| < \varepsilon \quad \forall m, n > N$

$= \frac{1}{m_1} + \frac{1}{m_2} + \ldots$