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3. The Fourier series for the function

$$f(x)=\begin{cases}1 & 0<x<1\\ -1 & -1<x<0\end{cases}$$

is

$$f(x)=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin(2n+1)\pi x$$

By carefully examining this Fourier series deduce the sum of the series

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\ldots\ldots$$

Choose $x=\frac{1}{2}$ $\quad 1=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\sin(2n+1)\frac{\pi}{2}=\frac{4}{\pi}\left\{\sin\frac{\pi}{2}+\frac{1}{3}\sin\frac{3\pi}{2}\ldots\right\}$

Hence $\boxed{1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots=\frac{\pi}{4}}$

Use the Fourier series of the function f(x) and the properties of Fourier series to obtain the Fourier series of

$$g(x) = \begin{cases} x & 0 < x < 1 \\ -x & -1 < x < 0 \end{cases}$$

(No marks will be awarded for a direct derivation of the Fourier series of g(x)).