2-22. The motion of a charged particle in an electromagnetic field can be obtained from the **Lorentz equation*** for the force on a particle in such a field. If the electric field vector is **E** and the magnetic field vector is **B**, the force on a particle of mass m that carries a charge q and has a velocity **v** is given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

where we assume that $v \ll c$ (speed of light).

(a) If there is no electric field and if the particle enters the magnetic field in a direction perpendicular to the lines of magnetic flux, show that the trajectory is a circle with radius

$$r = \frac{mv}{qB} = \frac{v}{\omega_c}$$

where $\omega_c \equiv qB/m$ is the *cyclotron frequency.*

(b) Choose the z-axis to lie in the direction of **B** and let the plane containing **E** and **B** be the yz–plane. Thus

$$\mathbf{B} = B\mathbf{k}, \quad \mathbf{E} = E_y\mathbf{j} + E_z\mathbf{k}$$

Show that the z component of the motion is given by

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m}t^2$$

where

$$z(0) \equiv z_0 \quad \text{and} \quad \dot{z}(0) \equiv \dot{z}_0$$

(c) Continue the calculation and obtain expressions for $\dot{x}(t)$ and $\dot{y}(t)$. Show that the time averages of these velocity components are

$$\langle \dot{x} \rangle = \frac{E_y}{B}, \quad \langle \dot{y} \rangle = 0$$

(Show that the motion is periodic and then average over one complete period.)

(d) Integrate the velocity equations found in (c) and show (with the initial conditions $x(0) = -A/\omega_c$, $\dot{x}(0) = E_y/B$, $y(0) = 0$, $\dot{y}(0) = A$) *that*

$$x(t) = \frac{-A}{\omega_c}\cos\omega_c t + \frac{E_y}{B}t, \quad y(t) = \frac{A}{\omega_c}\sin\omega_c t$$

These are the parametric equations of a trochoid. Sketch the projection of the trajectory on the xy–plane for the cases (i) $A > |E_y/B|$, (ii) $A < |E_y/B|$, and (iii) $A = |E_y/B|$. (The last case yields a cycloid.)