17

Nonparametric Methods:

Chi-Square Applications

GOALS

When you have completed this chapter, you will be able to:

1 List the characteristics of the *chi-square distribution.*

2 Conduct a test of hypothesis comparing an observed set of frequencies to an expected distribution.

3 Conduct a test of hypothesis to determine whether two classification criteria are related.

The quality control department at Food Town, Inc., a grocery chain in upstate New York, conducts a monthly check on the comparison of scanned prices to posted prices. The chart in Exercise 15 summarizes the results of a sample of 500 items last month. Company management would like to know whether there is any relationship between error rates on regularly priced items and specially priced items. Use the .01 significance level. (See Exercise 15 and Goal 3.)

Introduction

Chapters 9 through 12 discuss data of interval or ratio scale, such as weights of steel ingots, incomes of minorities, and years of employment. We conducted hypothesis tests about a single population mean, two population means, and three or more population means. For these tests we assume the populations follow the normal probability distribution. However, there are tests available in which no assumption regarding the shape of the population is necessary. These tests are referred to as nonparametric. This means the assumption of a normal population is not necessary.

There are also tests exclusively for data of nominal scale of measurement. Recall from Chapter 1 that nominal data is the "lowest" or most primitive. For this type of measurement, data are classified into categories where there is no natural order. Examples include gender of Congressional representatives, state of birth of students, or brand of peanut butter purchased. In this chapter we introduce a new test statistic, the chi-square statistic. We use it for data measured with a nominal scale.

Goodness-of-Fit Test: Equal Expected Frequencies

The goodness-of-fit test is one of the most commonly used statistical tests. The first illustration of this test involves the case in which the expected cell frequencies are equal.

As the full name implies, the purpose of the goodness-of-fit test is to compare an observed distribution to an expected distribution. An example will describe the hypothesis-testing situation.

Example

Ms. Jan Kilpatrick is the marketing manager for a manufacturer of sports cards. She plans to begin selling a series of cards with pictures and playing statistics of former Major League Baseball players. One of the problems is the selection of the former players. At a baseball card show at Southwyck Mall last weekend, she set up a booth and offered cards of the following six Hall of Fame baseball players: Tom Seaver, Nolan Ryan, Ty Cobb, George Brett, Hank Aaron, and Johnny Bench. At the end of the day she sold a total of 120 cards. The number of cards sold for each old-time player is shown in Table 17–1. Can she conclude the sales are not the same for each player?

TABLE 17–1 Number of Cards Sold for Each Player

Player	Cards Sold
Tom Seaver	13
Nolan Ryan	33
Ty Cobb	14
George Brett	7
Hank Aaron	36
Johnny Bench	17
Total	120

If there is no significant difference in the popularity of the players, we would expect that the observed frequencies (f_o) would be equal—or nearly equal. That is, we would expect to sell as many cards for Tom Seaver as for Nolan Ryan. Thus, any discrepancy in the observed and expected frequencies could be attributed to sampling (chance).

What about the level of measurement in this problem? Notice when a card is sold, the "measurement" of the card is based on the player's name. There is no natural order to the players. No one player is better than another. Therefore, a nominal scale is used to evaluate each observation.

Because there are 120 cards in the sample, we expect that 20 (f_e) cards, i.e., the expected frequency f_e, will fall in each of the six categories (Table 17–2). These categories are called **cells.** An examination of the set of observed frequencies in Table 17–1 indicates that the card for George Brett is sold rather infrequently, whereas the cards for Hank Aaron and Nolan Ryan are sold more often. Is the difference in sales due to chance, or can we conclude that there is a preference for the cards of certain players?

TABLE 17–2 Observed and Expected Frequencies for the 120 Cards Sold

Player	Cards Sold, f_o	Expected Number Sold, f_e
Tom Seaver	13	20
Nolan Ryan	33	20
Ty Cobb	14	20
George Brett	7	20
Hank Aaron	36	20
Johnny Bench	17	20
Total	120	120

Solution

We will use the same systematic five-step, hypothesis-testing procedure followed in previous chapters.

Step 1: State the null hypothesis and the alternate hypothesis. The null hypothesis, H_0, is that there is no difference between the set of observed frequencies and the set of expected frequencies; that is, any difference between the two sets of frequencies can be attributed to sampling (chance). The alternate hypothesis, H_1, is that there is a difference between the observed and expected sets of frequencies. If H_0 is rejected and H_1 is accepted, it means that sales are not equally distributed among the six categories (cells).

Step 2: Select the level of significance. We selected the .05 significance level. The probability is .05 that a true null hypothesis will be rejected.

Step 3: Select the test statistic. The test statistic follows the chi-square distribution, designated as χ^2:

CHI-SQUARE TEST STATISTIC

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] \quad \textbf{[17–1]}$$

with $k - 1$ degrees of freedom, where:

k is the number of categories.
f_o is an observed frequency in a particular category.
f_e is an expected frequency in a particular category.

We will examine the characteristics of the chi-square distribution in more detail shortly.

Statistics in Action

For many years, researchers and statisticians believed that all variables were normally distributed. In fact, it was generally assumed to be a universal law. However, Karl Pearson observed that experimental data were not always normally distributed but there was no way to prove his observations were correct. To solve this problem, Pearson discovered the chi-square statistic that basically compares an observed frequency distribution with an assumed normal distribution. His discovery proved that all variables were not normally distributed.

Step 4: Formulate the decision rule. Recall that the decision rule in hypothesis testing requires finding a number that separates the region where we do not reject H_0 from the region of rejection. This number is called the *critical value.* As we will soon see, the chi-square distribution is really a family of distributions. Each distribution has a slightly different shape, depending on the number of degrees of freedom. The number of degrees of freedom in this type of problem is found by $k - 1$, where k is the number of categories. In this particular problem there are six. Since there are six categories, there are $k - 1 = 6 - 1 = 5$ degrees of freedom. As noted, a category is called a *cell,* so there are six cells. The critical value for 5 degrees of freedom and the .05 level of significance is found in Appendix B.3. A portion of that table is shown in Table 17–3. The critical value is 11.070, found by locating 5 degrees of freedom in the left margin and then moving horizontally (to the right) and reading the critical value in the .05 column.

TABLE 17–3 A Portion of the Chi-Square Table

Degrees of Freedom *df*	Right-Tail Area .10	.05	.02	.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086

The decision rule is to reject H_0 if the computed value of chi-square is greater than 11.070. If it is less than or equal to 11.070, do not reject H_0. Chart 17–1 shows the decision rule.

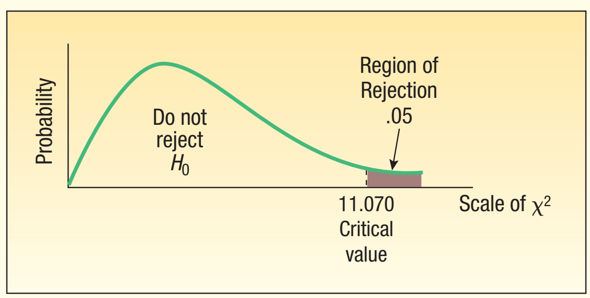


CHART 17–1 Chi-Square Probability Distribution for 5 Degrees of Freedom, Showing the Region of Rejection, .05 Level of Significance

The decision rule indicates that if there are large differences between the observed and expected frequencies, resulting in a computed χ^2 of more than 11.070, the null hypothesis should be rejected. However, if the differences between f_o and f_e are small, the computed χ^2 value will be 11.070 or less, and the null hypothesis should not be rejected. The reasoning is that such small differences between the observed and expected frequencies are probably due to chance. Remember, the 120 observations are a sample of the population.

Step 5: Compute the value of chi-square and make a decision. Of the 120 cards sold in the sample, we counted the number of times Tom Seaver and Nolan Ryan and each of the others were sold. The counts were reported in Table 17–1. The calculations for chi-square follow. (Note again that the expected frequencies are the same for each cell.)

Column 1: Determine the differences between each f_o and f_e. That is, $(f_o - f_e)$. The sum of these differences is zero.

Column 2: Square the difference between each observed and expected frequency, that is, $(f_o - f_e)^2$.

Column 3: Divide the result for each observation by the expected frequency. That is, $\frac{(f_o - f_e)^2}{f_e}$. Finally, sum these values.

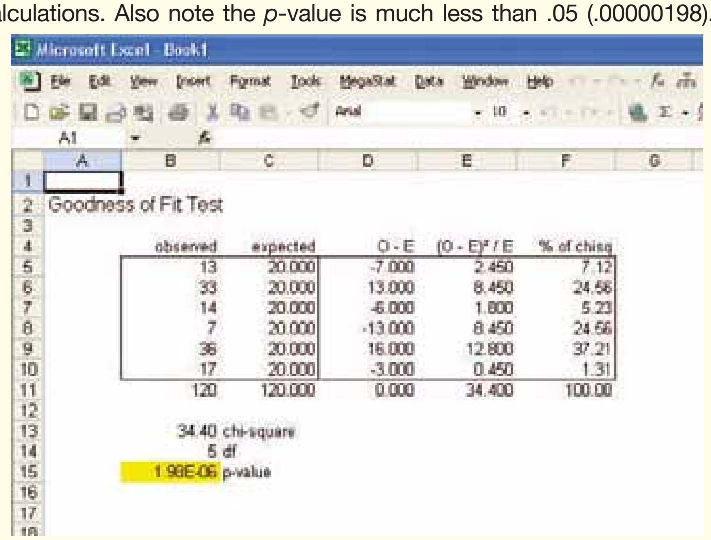
The result is the value of χ^2, which is 34.40.

Baseball Player	f_o	f_e	(1) $(f_o - f_e)$	(2) $(f_o - f_e)^2$	(3) $\frac{(f_o - f_e)^2}{f_e}$
Tom Seaver	13	20	−7	49	49/20 = 2.45
Nolan Ryan	33	20	13	169	169/20 = 8.45
Ty Cobb	14	20	−6	36	36/20 = 1.80
George Brett	7	20	−13	169	169/20 = 8.45
Hank Aaron	36	20	16	256	256/20 = 12.80
Johnny Bench	17	20	−3	9	9/20 = 0.45
			0		34.40
		Must be			χ^2

The computed χ^2 of 34.40 is in the rejection region beyond the critical value of 11.070. The decision, therefore, is to reject H_0 at the .05 level and to accept H_1. The difference between the observed and the expected frequencies is not due to chance. Rather, the differences between f_o and f_e are large enough to be considered significant. The chance these differences are due to sampling error is very small. So we conclude that it is unlikely that card sales are the same among the six players.

MegaStat

We can use software to compute the value of chi-square. The output of MegaStat follows. The steps are shown in the **Software Commands** section at the end of the chapter. The computed value of chi-square is 34.40, the same value obtained in our earlier calculations. Also note the p-value is much less than .05 (.00000198).



Goodness of Fit Test

observed	expected	O - E	(O - E)² / E	% of chisq
13	20.000	-7.000	2.450	7.12
33	20.000	13.000	8.450	24.56
14	20.000	-6.000	1.800	5.23
7	20.000	-13.000	8.450	24.56
36	20.000	16.000	12.800	37.21
17	20.000	-3.000	0.450	1.31
120	120.000	0.000	34.400	100.00

34.40 chi-square
5 df
1.98E-06 p-value

The chi-square distribution, which is used as the test statistic in this chapter, has the following characteristics.

1. **Chi-square values are never negative.** This is because the difference between f_o and f_e is squared, that is, $(f_o - f_e)^2$.
2. **There is a family of chi-square distributions.** There is a chi-square distribution for 1 degree of freedom, another for 2 degrees of freedom, another for 3 degrees of freedom, and so on. In this type of problem the number of degrees of freedom is determined by $k - 1$, where k is the number of categories. Therefore, the shape of the chi-square distribution does *not* depend on the size of the sample, but on the number of categories used. For example, if 200 employees of an airline were classified into one of three categories—flight personnel, ground support, and administrative personnel—there would be $k - 1 = 3 - 1 = 2$ degrees of freedom.
3. **The chi-square distribution is positively skewed.** However, as the number of degrees of freedom increases, the distribution begins to approximate the normal probability distribution. Chart 17–2 shows the distributions for selected degrees of freedom. Notice that for 10 degrees of freedom the curve is approaching a normal distribution.

Shape of χ^2 distribution approaches normal distribution as *df* becomes larger

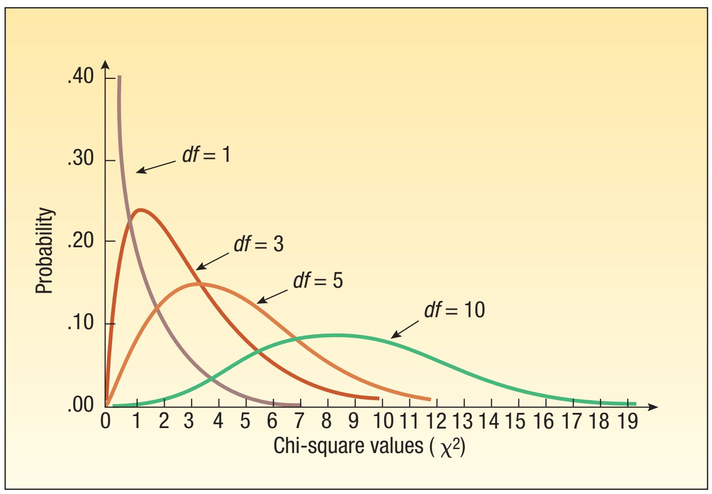


CHART 17–2 Chi-Square Distributions for Selected Degrees of Freedom

Self-Review 17–1

The human resources director at Georgetown Paper, Inc., is concerned about absenteeism among hourly workers. She decides to sample the company records to determine whether absenteeism is distributed evenly throughout the six-day work week. The hypotheses are:

H_0: Absenteeism is evenly distributed throughout the work week.
H_1: Absenteeism is *not* evenly distributed throughout the work week.

The sample results are:

	Number Absent		Number Absent
Monday	12	Thursday	10
Tuesday	9	Friday	9
Wednesday	11	Saturday	9

(a) What are the numbers 12, 9, 11, 10, 9, and 9 called?
(b) How many categories (cells) are there?
(c) What is the *expected* frequency for each day?
(d) How many degrees of freedom are there?
(e) What is the chi-square critical value at the 1 percent significance level?
(f) Compute the χ^2 test statistic.
(g) What is the decision regarding the null hypothesis?
(h) Specifically, what does this indicate to the human resources director?

Exercises

1. In a particular chi-square goodness-of-fit test there are four categories and 200 observations. Use the .05 significance level.
a. How many degrees of freedom are there?
b. What is the critical value of chi-square?

2. In a particular chi-square goodness-of-fit test there are six categories and 500 observations. Use the .01 significance level.
a. How many degrees of freedom are there?
b. What is the critical value of chi-square?

3. The null hypothesis and the alternate are:

H_0: The frequencies are equal.
H_1: The frequencies are not equal.

Category	f_o
A	10
B	20
C	30

a. State the decision rule, using the .05 significance level.
b. Compute the value of chi-square.
c. What is your decision regarding H_0?

4. The null hypothesis and the alternate are:

H_0: The frequencies are equal.
H_1: The frequencies are not equal.

Category	f_o
A	10
B	20
C	30
D	20

a. State the decision rule, using the .05 significance level.
b. Compute the value of chi-square.
c. What is your decision regarding H_0?

5. A six-sided die is rolled 30 times and the numbers 1 through 6 appear as shown in the following frequency distribution. At the .10 significance level, can we conclude that the die is fair?

Outcome	Frequency	Outcome	Frequency
1	3	4	3
2	6	5	9
3	2	6	7

6. Classic Golf, Inc., manages five courses in the Jacksonville, Florida, area. The director wishes to study the number of rounds of golf played per weekday at the five courses. He gathered the following sample information.

Day	Rounds
Monday	124
Tuesday	74
Wednesday	104
Thursday	98
Friday	120

At the .05 significance level, is there a difference in the number of rounds played by day of the week?

7. A group of department store buyers viewed a new line of dresses and gave their opinions of them. The results were:

Opinion	Number of Buyers	Opinion	Number of Buyers
Outstanding	47	Good	39
Excellent	45	Fair	35
Very good	40	Undesirable	34

Because the largest number (47) indicated the new line is outstanding, the head designer thinks that this is a mandate to go into mass production of the dresses. The head sweeper (who somehow became involved in this) believes that there is not a clear mandate and claims that the opinions are evenly distributed among the six categories. He further states that the slight differences among the various counts are probably due to chance. Test the null hypothesis that there is no significant difference among the opinions of the buyers. Test at the .01 level of risk. Follow a formal approach; that is, state the null hypothesis, the alternate hypothesis, and so on.

8. The safety director of Honda USA took samples at random from company records of minor work-related accidents and classified them according to the time the accident took place.

Time	Number of Accidents	Time	Number of Accidents
8 up to 9 A.M.	6	1 up to 2 P.M.	7
9 up to 10 A.M.	6	2 up to 3 P.M.	8
10 up to 11 A.M.	20	3 up to 4 P.M.	19
11 up to 12 P.M.	8	4 up to 5 P.M.	6

Using the goodness-of-fit test and the .01 level of significance, determine whether the accidents are evenly distributed throughout the day. Write a brief explanation of your conclusion.

Goodness-of-Fit Test: Unequal Expected Frequencies

The expected frequencies (f_e) in the previous distribution involving baseball cards were all equal (20). According to the null hypothesis, it was expected that a picture of Tom Seaver would sell 20 times at random, a picture of Johnny Bench would sell 20 times out of 120 trials, and so on. The chi-square test can also be used if the expected frequencies are not equal.

Expected frequencies not equal in this problem

The following example illustrates the case of unequal frequencies and also gives a practical use of the chi-square goodness-of-fit test—namely, to find whether a local experience differs from the national experience.

Example

The American Hospital Administrators Association (AHAA) reports the following information concerning the number of times senior citizens are admitted to a hospital during a one-year period. Forty percent are not admitted; 30 percent are admitted once; 20 percent are admitted twice, and the remaining 10 percent are admitted three or more times.

A survey of 150 residents of Bartow Estates, a community devoted to active seniors located in central Florida, revealed 55 residents were not admitted during the last year, 50 were admitted to a hospital once, 32 were admitted twice, and the rest of those in the survey were admitted three or more times. Can we conclude the survey at Bartow Estates is consistent with the information suggested by the AHAA? Use the .05 significance level.

Solution

We begin by organizing the above information into Table 17–4. Clearly, we cannot compare percentages given in the study by the Hospital Administrators to the frequencies reported for the Bartow Estates. However, these percentages can be converted to expected frequencies, f_e. According to the Hospital Administrators, 40 percent of the Bartow residents in the survey did not require hospitalization. Thus, if there is no difference between the national experience and those of Bartow Estates, then 40 percent of the 150 seniors surveyed (60 residents) would not have been hospitalized. Further, 30 percent of those surveyed were admitted once (45 residents), and so on. The observed frequencies for Bartow residents and the expected frequencies based on the percents in the national study are given in Table 17–4.

Statistics in Action

Many state governments operate lotteries to help fund education. In many lotteries, numbered balls are mixed and selected by a machine. In a Select Three game, numbered balls are selected randomly from three groups of balls numbered zero through nine. Randomness would predict that the frequency of each number is equal. How would you prove that the selection machine ensured randomness? A chi-square, goodness-of-fit test could be used to prove or disprove randomness.

TABLE 17–4 Summary of Study by AHAA and a Survey of Bartow Estates Residents

Number of Times Admitted	AHAA Percent of Total	Number of Bartow Residents (f_o)	Expected Number of Residents (f_e)
0	40	55	60
1	30	50	45
2	20	32	30
3 or more	10	13	15
Total	100	150	150

The null hypothesis and the alternate hypothesis are:

H_0: There is no difference between local and national experience for hospital admissions.
H_1: There is a difference between local and national experience for hospital admissions.

To find the decision rule we use Appendix B.3 and the .05 significance level. There are four admitting categories, so the degrees of freedom are $df = 4 - 1 = 3$. The critical value is 7.815. Therefore, the decision rule is to reject the null hypothesis if $\chi^2 > 7.815$. The decision rule is portrayed in Chart 17–3.

Now to compute the chi-square test statistic:

Number of Times Admitted	(f_o)	(f_e)	$f_o - f_e$	$(f_o - f_e)^2/f_e$
0	55	60	−5	0.4167
1	50	45	5	0.5556
2	32	30	2	0.1333
3 or more	13	15	−2	0.2667
Total	150	150	0	1.3723

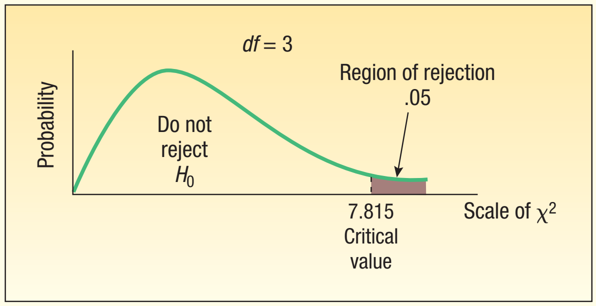


CHART 17–3 Decision Criteria for the Bartow Estates Research Study

The computed value of χ^2 (1.3723) lies to the left of 7.815. Thus, we cannot reject the null hypothesis. We conclude that there is no evidence of a difference between the local and national experience for hospital admissions.

Limitations of Chi-Square

Be careful in applying χ^2 to some problems.

If there is an unusually small expected frequency in a cell, chi-square (if applied) might result in an erroneous conclusion. This can happen because f_e appears in the denominator, and dividing by a very small number makes the quotient quite large! Two generally accepted policies regarding small cell frequencies are:

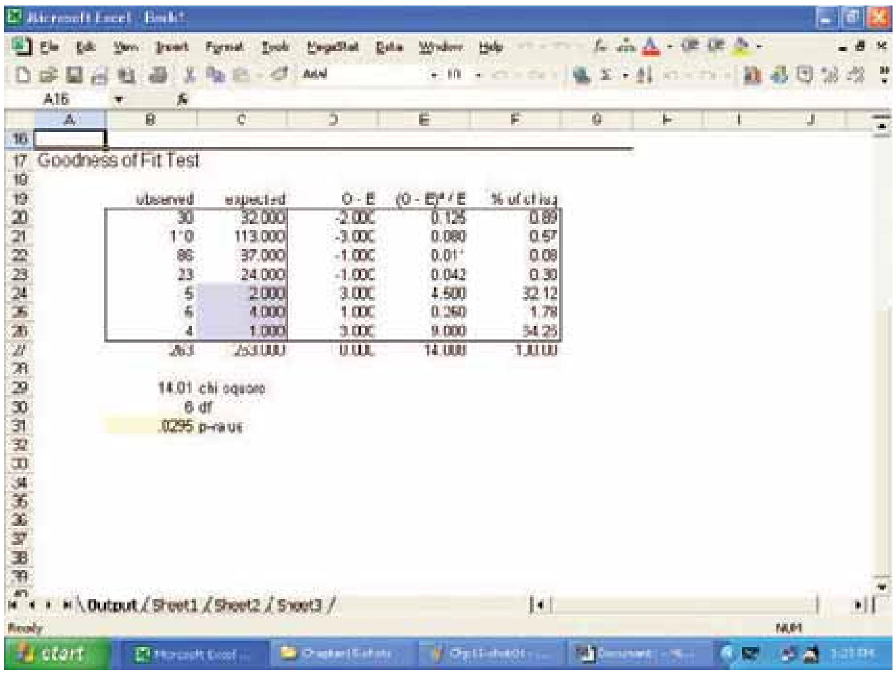
1. If there are only two cells, the *expected* frequency in each cell should be at least 5. The computation of chi-square would be permissible in the following problem, involving a minimum f_e of 6.

Individual	f_o	f_e
Literate	643	642
Illiterate	7	6

2. For more than two cells, chi-square should *not* be used if more than 20 percent of the f_e cells have expected frequencies less than 5. According to this policy, it would not be appropriate to use the goodness-of-fit test on the following data. Three of the seven cells, or 43 percent, have expected frequencies (f_e) of less than 5.

Level of Management	f_o	f_e
Foreman	30	32
Supervisor	110	113
Manager	86	87
Middle management	23	24
Assistant vice president	5	2
Vice president	5	4
Senior vice president	4	1
Total	263	263

To show the reason for the 20 percent policy, we conducted the goodness-of-fit test on the above data on the levels of management. The MegaStat output follows.

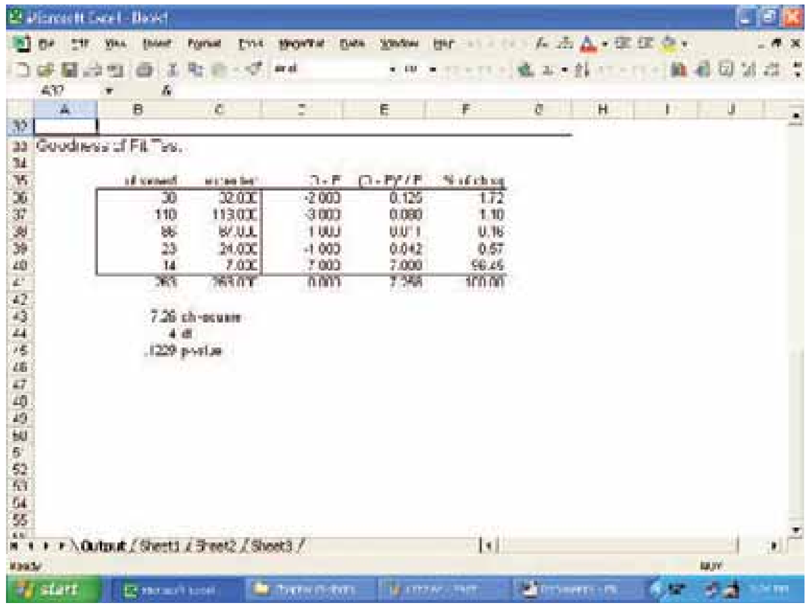


For this test at the .05 significance level, H_0 is rejected if the computed value of chi-square is greater than 12.592. The computed value is 14.01, so we reject the null hypothesis that the observed frequencies represent a random sample from the population of the expected values. Examine the MegaStat output. More than 98 percent of the computed chi-square value is accounted for by the three vice president categories ([4.500 + .250 + 9.000]/14.008 = 0.9815). Logically, too much weight is being given to these categories.

The dilemma can be resolved by combining categories if it is logical to do so. In the above example we combine the three vice presidential categories, which satisfies the 20 percent policy.

Level of Management	f_o	f_e
Foreman	30	32
Supervisor	110	113
Manager	86	87
Middle management	23	24
Vice president	14	7
Total	263	263

The computed value of chi-square with the revised categories is 7.26. See the following MegaStat output. This value is less than the critical value of 9.488 for the .05 significance level. The null hypothesis is, therefore, not rejected at the .05 significance level. This indicates there is not a significant difference between the observed distribution and the expected distribution.



Self-Review 17–2

The American Accounting Association classifies accounts receivable as "current," "late," and "not collectible." Industry figures show that 60 percent of accounts receivable are current, 30 percent are late, and 10 percent are not collectible. Massa and Barr, a law firm in Greenville, Ohio, has 500 accounts receivable: 320 are current, 120 are late, and 60 are not collectible. Are these numbers in agreement with the industry distribution? Use the .05 significance level.

Exercises

9. The following hypotheses are given:

H_0: Forty percent of the observations are in category A, 40 percent are in B, and 20 percent are in C.
H_1: The distribution of the observations is not as described in H_0.

We took a sample of 60, with the following results.

Category	f_o
A	30
B	20
C	10

a. State the decision rule using the .01 significance level.
b. Compute the value of chi-square.
c. What is your decision regarding H_0?

10. The chief of security for Mall of the Dakotas was directed to study the problem of missing goods. He selected a sample of 100 boxes that had been tampered with and ascertained that, for 60 of the boxes, the missing pants, shoes, and so on were attributed to shoplifting. For 30 other boxes employees had stolen the goods, and for the remaining 10 boxes he blamed poor inventory control. In his report to the mall management, can he say that shoplifting is *twice* as likely to be the cause of the loss as compared with either employee theft or poor inventory control and that employee theft and poor inventory control are equally likely? Use the .02 significance level.

11. The bank credit card department of Carolina Bank knows from experience that 5 percent of its card holders have had some high school, 15 percent have completed high school, 25 percent have had some college, and 55 percent have completed college. Of the 500 card holders whose cards have been called in for failure to pay their charges this month, 50 had some high school, 100 had completed high school, 190 had some college, and 160 had completed college. Can we conclude that the distribution of card holders who do not pay their charges is different from all others? Use the .01 significance level.

12. For many years TV executives used the guideline that 30 percent of the audience were watching each of the traditional big three prime-time networks and 10 percent were watching cable stations on a weekday night. A random sample of 500 viewers in the Tampa–St. Petersburg, Florida, area last Monday night showed that 165 homes were tuned in to the ABC affiliate, 140 to the CBS affiliate, 125 to the NBC affiliate, and the remainder were viewing a cable station. At the .05 significance level, can we conclude that the guideline is still reasonable?

Contingency Table Analysis

In Chapter 4 we discussed bivariate data, where we studied the relationship between two variables. We described a contingency table, which simultaneously summarizes two nominal-scale variables of interest. For example, a sample of students enrolled in the School of Business is classified by gender (male or female) and major (accounting, management, finance, marketing, or quantitative methods). This classification is based on the nominal scale, because there is no natural order to the classifications.

We discussed contingency tables in Chapter 5. On page 156, we illustrated the relationship between loyalty to a company and the length of employment and explored whether older employees were likely to be more loyal to the company.

We can use the chi-square statistic to formally test for a relationship between two nominal-scaled variables. To put it another way, Is one variable *independent* of the other? Here are some examples where we are interested in testing whether two variables are related.

- Ford Motor Company operates an assembly plant in Dearborn, Michigan. The plant operates three shifts per day, 5 days a week. The quality control manager wishes to compare the quality level on the three shifts. Vehicles are classified by quality level (acceptable, unacceptable) and shift (day, afternoon, night). Is there a difference in the quality level on the three shifts? That is, is the quality of the product related to the shift when it was manufactured? Or is the quality of the product independent of the shift on which it was manufactured?
- A sample of 100 drivers who were stopped for speeding violations was classified by gender and whether or not they were wearing a seat belt. For this sample, is wearing a seatbelt related to gender?
- Does a male released from federal prison make a different adjustment to civilian life if he returns to his hometown or if he goes elsewhere to live? The two variables are adjustment to civilian life and place of residence. Note that both variables are measured on the nominal scale.

Example

The Federal Correction Agency is investigating the last question cited above: Does a male released from federal prison make a different adjustment to civilian life if he returns to his hometown or if he goes elsewhere to live? To put it another way, is there a relationship between adjustment to civilian life and place of residence after release from prison? Use the .01 significance level.

Solution

As before, the first step in hypothesis testing is to state the null and alternate hypotheses.

H_0: There is no relationship between adjustment to civilian life and where the individual lives after being released from prison.
H_1: There is a relationship between adjustment to civilian life and where the individual lives after being released from prison.

The agency's psychologists interviewed 200 randomly selected former prisoners. Using a series of questions, the psychologists classified the adjustment of each individual to civilian life as outstanding, good, fair, or unsatisfactory. The classifications for the 200 former prisoners were tallied as follows. Joseph Camden, for example, returned to his hometown and has shown outstanding adjustment to civilian life. His case is one of the 27 tallies in the upper left box.

Residence after Release from Prison	Adjustment to Civilian Life			
	Outstanding	Good	Fair	Unsatisfactory
Hometown	𝍸 𝍸 𝍸 𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	𝍸 𝍸 𝍸 𝍸 𝍸
Not hometown	𝍸 𝍸 |||	𝍸 𝍸 𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸

Contingency table consists of count data.

The tallies in each box, or *cell,* were counted. The counts are given in the following **contingency table.** (See Table 17–5.) In this case, the Federal Correction Agency wondered whether adjustment to civilian life is *contingent on* where the prisoner goes after release from prison.

TABLE 17–5 Adjustment to Civilian Life and Place of Residence

Residence after Release from Prison	Adjustment to Civilian Life				
	Outstanding	Good	Fair	Unsatisfactory	Total
Hometown	27	35	33	25	120
Not hometown	13	15	27	25	80
Total	40	50	60	50	200

Once we know how many rows (2) and columns (4) there are in the contingency table, we can determine the critical value and the decision rule. For a chi-square test of significance where two traits are classified in a contingency table, the degrees of freedom are found by:

$$df = (\text{number of rows} - 1)(\text{number of columns} - 1) = (r - 1)(c - 1)$$

In this problem:

$$df = (r - 1)(c - 1) = (2 - 1)(4 - 1) = 3$$

To find the critical value for 3 degrees of freedom and the .01 level (selected earlier), refer to Appendix B.3. It is 11.345. The decision rule is to reject the null hypothesis if the computed value of χ^2 is greater than 11.345. The decision rule is portrayed graphically in Chart 17–4.

Statistics in Action

A study of 1,000 Americans over the age of 24 showed that 28 percent never married. Of those, 22 percent completed college. Twenty-three percent of the 1,000 married and completed college. Can we conclude for the information given that being married is related to completing college? The study indicated that the two variables were related, that the computed value of the chi-square statistic was 9.368, and the *p*-value was .002. Can you duplicate these results?

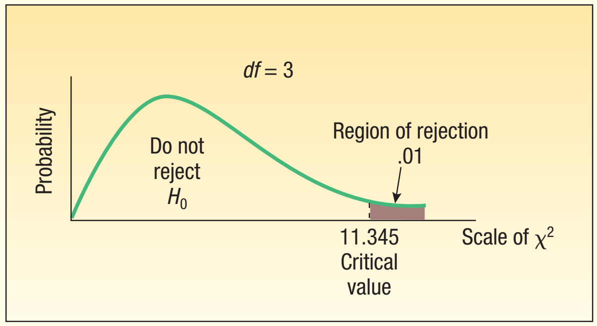


CHART 17–4 Chi-Square Distribution for 3 Degrees of Freedom

Next we find the computed value of χ^2. The observed frequencies, f_o, are shown in Table 17–5. How are the corresponding expected frequencies, f_e, determined? Note in the "Total" column of Table 17–5 that 120 of the 200 former prisoners (60 percent) returned to their hometowns. *If there were no relationship* between adjustment and residency after release from prison, we would expect 60 percent of the 40 ex-prisoners who made outstanding adjustment to civilian life to reside in their hometowns. Thus, the expected frequency f_e for the upper left cell is $.60 \times 40 = 24$. Likewise, if there were no relationship between adjustment and present residence, we would expect 60 percent of the 50 ex-prisoners (30) who had "good" adjustment to civilian life to reside in their hometowns.

Further, notice that 80 of the 200 ex-prisoners studied (40 percent) did not return to their hometowns to live. Thus, of the 60 considered by the psychologists to have made "fair" adjustment to civilian life, $.40 \times 60$, or 24, would be expected not to return to their hometowns.

The expected frequency for any cell can be determined by

EXPECTED FREQUENCY
$$f_e = \frac{\text{(Row total)(Column total)}}{\text{Grand total}} \qquad \textbf{[17–2]}$$

From this formula, the expected frequency for the upper left cell in Table 17–5 is:

$$\text{Expected frequency} - \frac{\text{(Row total)(Column total)}}{\text{Grand total}} = \frac{(120)(40)}{200} = 24$$

The observed frequencies, f_o, and the expected frequencies, f_e, for all of the cells in the contingency table are listed in Table 17–6.

TABLE 17–6 Observed and Expected Frequencies

Residence after Release from Prison	Adjustment to Civilian Life: Outstanding f_o	Outstanding f_e	Good f_o	Good f_e	Fair f_o	Fair f_e	Unsatisfactory f_o	Unsatisfactory f_e	Total f_o	Total f_e
Hometown	27	24	35	30	33	36	25	30	120	120
Not hometown	13	16	15	20	27	24	25	20	80	80
Total	40	40	50	50	60	60	50	50	200	200

Must be equal

$\frac{(80)(50)}{200}$

Must be equal

Recall that the computed value of chi-square using formula (17–1) is found by:

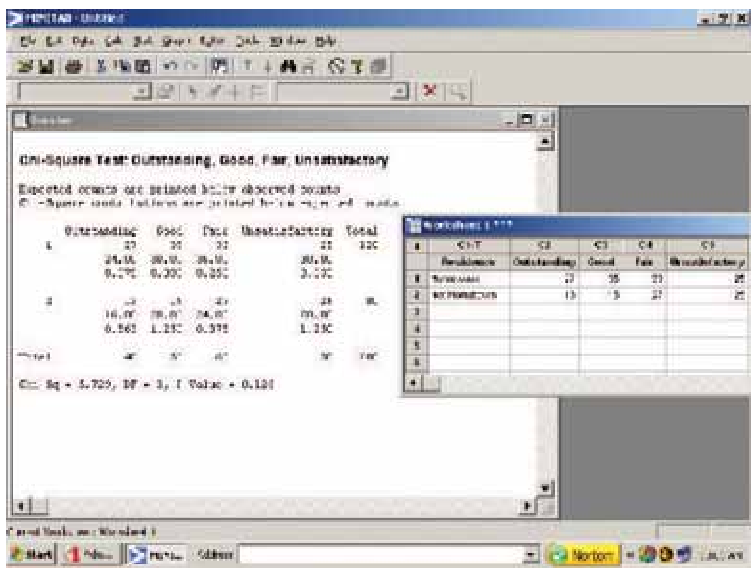
$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right]$$

Starting with the upper left cell:

$$\chi^2 = \frac{(27-24)^2}{24} + \frac{(35-30)^2}{30} + \frac{(33-36)^2}{36} + \frac{(25-30)^2}{30}$$
$$+ \frac{(13-16)^2}{16} + \frac{(15-20)^2}{20} + \frac{(27-24)^2}{24} + \frac{(25-20)^2}{20}$$
$$= 0.375 + 0.833 + 0.250 + 0.833 + 0.563 + 1.250 + 0.375 + 1.250$$
$$= 5.729$$

Because the computed value of chi-square (5.729) lies in the region to the left of 11.345, the null hypothesis is not rejected at the .01 significance level. We conclude there is no evidence of a relationship between adjustment to civilian life and where the prisoner resides after being released from prison. For the Federal Correction Agency's advisement program, adjustment to civilian life is not related to where the ex-prisoner lives.

The following output is from the MINITAB system.



Observe that the value of chi-square is the same as that computed earlier. In addition, the p-value is reported, .126. So the probability of finding a value of the test statistic as large or larger is .126 when the null hypothesis is true. The p-value also results in the same decision, do not reject the null hypothesis.

Self-Review 17–3

A social scientist sampled 140 people and classified them according to income level and whether or not they played a state lottery in the last month. The sample information is reported below. Is it reasonable to conclude that playing the lottery is related to income level? Use the .05 significance level.

	Income			
	Low	Middle	High	Total
Played	46	28	21	95
Did not play	14	12	19	45
Total	60	40	40	140

(a) What is this table called?
(b) State the null hypothesis and the alternate hypothesis.
(c) What is the decision rule?
(d) Determine the value of chi-square.
(e) Make a decision on the null hypothesis. Interpret the result.

Exercises

13. The director of advertising for the *Carolina Sun Times,* the largest newspaper in the Carolinas, is studying the relationship between the type of community in which a subscriber resides and the section of the newspaper he or she reads first. For a sample of readers, she collected the following sample information.

	National News	Sports	Comics
City	170	124	90
Suburb	120	112	100
Rural	130	90	88

At the .05 significance level, can we conclude there is a relationship between the type of community where the person resides and the section of the paper read first?

14. Four brands of lightbulbs are being considered for use in the final assembly area of the Saturn plant in Spring Hill, Tennessee. The director of purchasing asked for samples of 100 from each manufacturer. The numbers of acceptable and unacceptable bulbs from each manufacturer are shown below. At the .05 significance level, is there a difference in the quality of the bulbs?

	Manufacturer			
	A	B	C	D
Unacceptable	12	8	5	11
Acceptable	88	92	95	89
Total	100	100	100	100

15. The quality control department at Food Town, Inc., a grocery chain in upstate New York, conducts a monthly check on the comparison of scanned prices to posted prices. The chart below summarizes the results of a sample of 500 items last month. Company management would like to know whether there is any relationship between error rates on regularly priced items and specially priced items. Use the .01 significance level.

	Regular Price	Advertised Special Price
Undercharge	20	10
Overcharge	15	30
Correct price	200	225

16. The use of cellular phones in automobiles has increased dramatically in the last few years. Of concern to traffic experts, as well as manufacturers of cellular phones, is the effect on accident rates. Is someone who is using a cellular phone more likely to be involved in a traffic accident? What is your conclusion from the following sample information? Use the .05 significance level.

	Had Accident in the Last Year	Did Not Have an Accident in the Last Year
Uses a cell phone	25	300
Does not use a cell phone	50	400

Chapter Summary

I. The characteristics of the chi-square distribution are:
  - **A.** The value of chi-square is never negative.
  - **B.** The chi-square distribution is positively skewed.
  - **C.** There is a family of chi-square distributions.
    - **1.** Each time the degrees of freedom change, a new distribution is formed.
    - **2.** As the degrees of freedom increase, the distribution approaches a normal distribution.

II. A goodness-of-fit test will show whether an observed set of frequencies could have come from a hypothesized population distribution.
  - **A.** The degrees of freedom are $k - 1$, where k is the number of categories.
  - **B.** The formula for computing the value of chi-square is

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] \quad \textbf{[17–1]}$$

III. A contingency table is used to test whether two traits or characteristics are related.
  - **A.** Each observation is classified according to two traits.
  - **B.** The expected frequency is determined as follows:

$$f_e = \frac{\text{(Row total)(Column total)}}{\text{Grand total}} \quad \textbf{[17–2]}$$

  - **C.** The degrees of freedom are found by:

$$df = (\text{Rows} - 1)(\text{Columns} - 1)$$

  - **D.** The usual hypothesis testing procedure is used.

Pronunciation Key

SYMBOL	MEANING	PRONUNCIATION
χ^2	Probability distribution	*ki square*
f_o	Observed frequency	*f sub oh*
f_e	Expected frequency	*f sub e*

Chapter Exercises

17. Vehicles heading west on Front Street may turn right, left, or go straight ahead at Elm Street. The city traffic engineer believes that half of the vehicles will continue straight through the intersection. Of the remaining half, equal proportions will turn right and left.

Two hundred vehicles were observed, with the following results. Can we conclude that the traffic engineer is correct? Use the .10 significance level.

	Straight	Right Turn	Left Turn
Frequency	112	48	40

18. The publisher of a sports magazine plans to offer new subscribers one of three gifts: a sweatshirt with the logo of their favorite team, a coffee cup with the logo of their favorite team, or a pair of earrings also with the logo of their favorite team. In a sample of 500 new subscribers, the number selecting each gift is reported below. At the .05 significance level, is there a preference for the gifts or should we conclude that the gifts are equally well liked?

Gift	Frequency
Sweatshirt	183
Coffee cup	175
Earrings	142

19. In a particular market there are three commercial television stations, each with its own evening news program from 6:00 to 6:30 P.M. According to a report in this morning's local newspaper, a random sample of 150 viewers last night revealed 53 watched the news on WNAE (channel 5), 64 watched on WRRN (channel 11), and 33 on WSPD (channel 13). At the .05 significance level, is there a difference in the proportion of viewers watching the three channels?

20. There are four entrances to the Government Center Building in downtown Philadelphia. The building maintenance supervisor would like to know if the entrances are equally utilized. To investigate, 400 people were observed entering the building. The number using each entrance is reported below. At the .01 significance level, is there a difference in the use of the four entrances?

Entrance	Frequency
Main Street	140
Broad Street	120
Cherry Street	90
Walnut Street	50
Total	400

21. The owner of a mail-order catalog would like to compare her sales with the geographic distribution of the population. According to the United States Bureau of the Census, 21 percent of the population lives in the Northeast, 24 percent in the Midwest, 35 percent in the South, and 20 percent in the West. Listed below is a breakdown of a sample of 400 orders randomly selected from those shipped last month. At the .01 significance level, does the distribution of the orders reflect the population?

Region	Frequency
Northeast	68
Midwest	104
South	155
West	73
Total	400

22. Banner Mattress and Furniture Company wishes to study the number of credit applications received per day for the last 300 days. The information is reported on the next page.

Number of Credit Applications	Frequency (Number of Days)
0	50
1	77
2	81
3	48
4	31
5 or more	13

To interpret, there were 50 days on which no credit applications were received, 77 days on which only one application was received, and so on. Would it be reasonable to conclude that the population distribution is Poisson with a mean of 2.0? Use the .05 significance level. *Hint:* To find the expected frequencies use the Poisson distribution with a mean of 2.0. Find the probability of exactly one success given a Poisson distribution with a mean of 2.0. Multiply this probability by 300 to find the expected frequency for the number of days in which there was exactly one application. Determine the expected frequency for the other days in a similar manner.

23. In the early 2000s, Deep Down Mining Company implemented new safety guidelines. Prior to these new guidelines, management expected there to be no accidents in 40 percent of the months, one accident in 30 percent of the months, two accidents in 20 percent of the months, and three accidents in 10 percent of the months. Over the last 10 years, or 120 months, there have been 46 months in which there were no accidents, 40 months in which there was one accident, 22 months in which there were two accidents, and 12 months in which there were 3 accidents. At the .05 significance level can the management at Deep Down conclude that there has been a change in the monthly accident distribution?

24. In 2005, John G. Roberts was nominated by President Bush and confirmed by the Senate to be the 17th U.S. Supreme Court Chief Justice. During the nomination process, John G. Roberts's career as a lawyer and judge was the subject of many studies. For example, Kenneth Manning, associate professor of political science at the University of Massachusetts–Dartmouth presented a research paper titled "How Right Is He?" at the 2005 meeting of the American Political Science Association. In cases where Judge Roberts participated, the study categorized the cases into three types: criminal justice, civil rights, and economic activity. In each case, the study also labeled Judge Roberts's vote as either liberal or conservative. Forty-five cases that could not be clearly categorized for the study were not included. At the 0.01 significance level can we conclude Judge Roberts is more conservative in some types of cases?

	Criminal Justice	Civil Rights	Economic Activity
Liberal	6	2	39
Conservative	38	11	49

25. A survey by *USA Today* investigated the public's attitude toward the federal deficit. Each sampled citizen was classified as to whether they felt the government should reduce the deficit, increase the deficit, or if they had no opinion. The sample results of the study by gender are reported below.

Gender	Reduce the Deficit	Increase the Deficit	No Opinion
Female	244	194	68
Male	305	114	25

At the .05 significance level, is it reasonable to conclude that gender is independent of a person's position on the deficit?

26. A study regarding the relationship between age and the amount of pressure sales personnel feel in relation to their jobs revealed the following sample information. At the .01 significance level, is there a relationship between job pressure and age?

	Degree of Job Pressure		
Age (years)	**Low**	**Medium**	**High**
Less than 25	20	18	22
25 up to 40	50	46	44
40 up to 60	58	63	59
60 and older	34	43	43

27. The claims department at Wise Insurance Company believes that younger drivers have more accidents and, therefore, should be charged higher insurance rates. Investigating a sample of 1,200 Wise policyholders revealed the following breakdown on whether a claim had been filed in the last three years and the age of the policyholder. Is it reasonable to conclude that there is a relationship between the age of the policyholder and whether or not the person filed a claim? Use the .05 significance level.

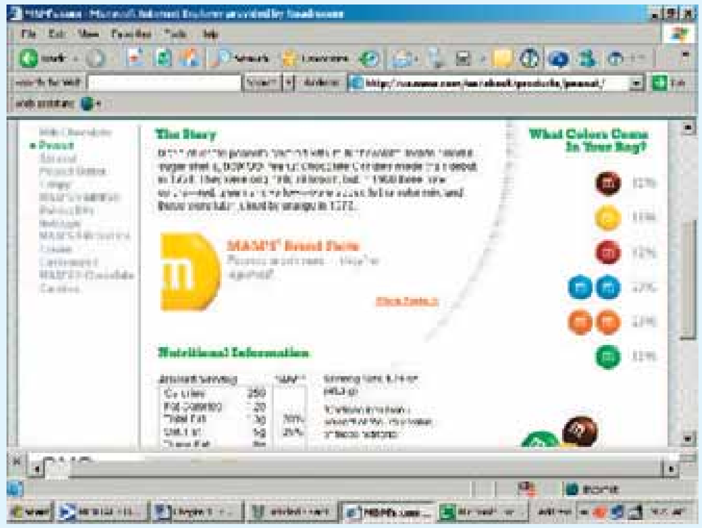
Age Group	No Claim	Claim
16 up to 25	170	74
25 up to 40	240	58
40 up to 55	400	44
55 or older	190	24
Total	1,000	200

28. A sample of employees at a large chemical plant was asked to indicate a preference for one of three pension plans. The results are given in the following table. Does it seem that there is a relationship between the pension plan selected and the job classification of the employees? Use the .01 significance level.

	Pension Plan		
Job Class	**Plan A**	**Plan B**	**Plan C**
Supervisor	10	13	29
Clerical	19	80	19
Labor	81	57	22

exercises.com

29. Did you ever purchase a bag of M&M's candies and wonder about the distribution of colors? You can go to the website www.baking.m-ms.com and click the United States on the map, then click **About M&M's,** then **History of M&M's Brand, Product Information,** and **Peanut** and find the percentage breakdown according to the manufacturer, as well as a brief history of the product. Did you know in the beginning they were all brown? For peanut M&M's 12 percent are brown, 15 percent yellow, 12 percent red, 23 percent blue, 23 percent orange, and 15 percent green. A 6-oz. bag purchased at the Book Store at Coastal Carolina University on November 1, 2005, had 12 blue, 14 brown, 13 yellow, 14 red, 7 orange, and 12 green. Is it reasonable to conclude that the actual distribution agrees with the expected distribution? Use the .05 significance level. Conduct your own trial. Be sure to share with your instructor.



30. As described in earlier chapters, many real estate companies and rental agencies now publish their listings on the World Wide Web. One example is Dunes Realty Company located in Garden City, South Carolina, and Surfside Beach, South Carolina. Go to the website http://www.dunes.com and click on **Vacation Rentals** and then **Beach Home Search,** then indicate at least 5 bedrooms, accommodations for at least 14 people, oceanfront, and no pool or floating dock; select a period in March; indicate that you are willing to spend up to $8,000 per week; and finally click on **Search the Beach Homes.** Sort the cottages offered into a contingency table by the number of bathrooms and whether the rental price is less than $2,000 for the week or $2,000 or more. You may need to combine some of the cells. Conduct a statistical test to determine if the number of bedrooms is related to the cost. Use the .05 significance level.

Data Set Exercises

31. Refer to the Real Estate data, which report information on homes sold in the Denver, Colorado, area last year.
- **a.** Develop a contingency table that shows whether a home has a pool and the township in which the house is located. Is there an association between the variables pool and township? Use the .05 significance level.
- **b.** Develop a contingency table that shows whether a home has an attached garage and the township in which the home is located. Is there an association between the variables attached garage and township? Use the .05 significance level.

32. Refer to the Baseball 2005 data, which report information on the 30 Major League Baseball teams for the 2005 season. Set up a variable that divides the teams into two groups, those that had a winning season and those that did not. There are 162 games in the season, so define a winning season as having won 81 or more games. Next, divide the teams into two salary groups. Let the 15 teams with the largest salaries be in one group and the 15 teams with the smallest salaries in the other. At the .05 significance level is there a relationship between salaries and winning?

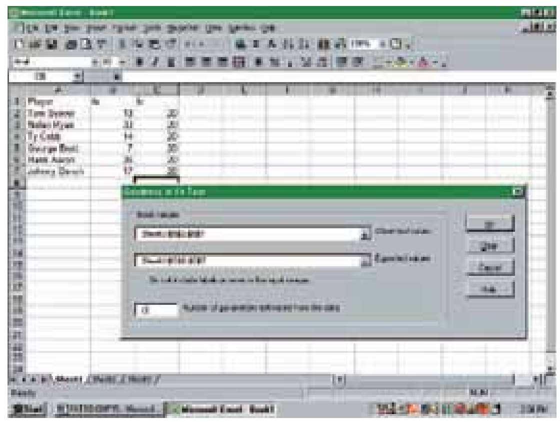
33. Refer to the Wage data, which report information on annual wages for a sample of 100 workers. Also included are variables relating to industry, years of education, and gender for each worker. Develop a table showing the industry of employment by gender. At the .05 significance level is it reasonable to conclude that industry of employment and gender are related?

34. Refer to the CIA data, which report demographic and economic information on 46 countries.
- **a.** Develop a contingency table that shows G-20 membership versus level of petroleum activity. Is there a significant association at the .05 level of significance between these variables?

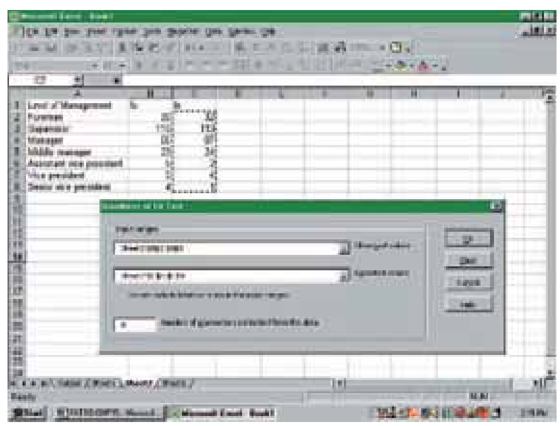
b. Group the countries into "young" (percent of population over 65 is less than 10) and "old" (percent of population over 65 is more than 10). Then develop a contingency table between this age variable and the level of petroleum activity. At the .05 level of significance can we conclude these variables are related?

Software Commands

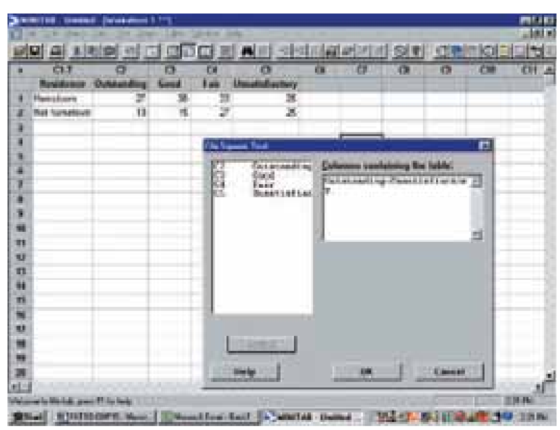
1. The MegaStat commands to create the chi-square goodness-of-fit test on page 650 are:
   a. Enter the information from Table 17–2 into a worksheet as shown.
   b. Select **MegaStat, Chi-Square/Crosstabs,** and **Goodness of Fit Test** and hit **Enter.**
   c. In the dialog box select *B2:B7* as the **Observed values,** *C2:C7* as the **Expected values,** and enter *0* as the **Number of parameters estimated from the data.** Click on **OK.**



2. The MegaStat commands to create the chi-square goodness-of-fit tests on pages 656 and 657 are the same except for the number of items in the observed and expected frequency columns. Only one dialog box is shown.
   a. Enter the Levels of Management information shown on page 655.
   b. Select **MegaStat, Chi-Square/Crosstabs,** and **Goodness of Fit Test** and hit **Enter.**
   c. In the dialog box select *B2:B8* as the **Observed values,** *C2:C8* as the **Expected values,** and enter *0* as the **Number of parameters estimated from the data.** Click on **OK.**



3. The MINITAB commands for the chi-square analysis on page 661 are:
   a. Enter the names of the variables in the first row and the data in the next two rows.
   b. Select **Stat, Table,** and then click on **Chi-Square Test** and hit **Enter.**
   c. In the dialog box select the columns labeled *Outstanding* to *Unsatisfactory* and click **OK.**



Chapter 17 Answers to Self-Review

17–1 **a.** Observed frequencies.
b. Six (six days of the week).
c. 10. Total observed frequencies ÷ 6 = 60/6 = 10.
d. 5; $k - 1 = 6 - 1 = 5$.
e. 15.086 (from the chi-square table in Appendix B.3).
f.

$$\chi^2 = \Sigma\left[\frac{(f_o - f_e)^2}{f_e}\right] = \frac{(12-10)^2}{10} + \cdots + \frac{(9-10)^2}{10} = 0.8$$

g. Do not reject H_0.
h. Absenteeism is distributed evenly throughout the week. The observed differences are due to sampling variation.

17–2 H_0: $P_C = .60$, $P_L = .30$, and $P_U = .10$.
H_1: Distribution is not as above.
Reject H_0 if $\chi^2 > 5.991$.

Category	f_o	f_e	$\frac{(f_o - f_e)^2}{f_e}$
Current	320	300	1.33
Late	120	150	6.00
Uncollectible	60	50	2.00
	500	500	9.33

Reject H_0. The accounts receivable data does not reflect the national average.

17–3 **a.** Contingency table
b. H_0: There is no relationship between income and whether the person played the lottery. H_1: There is a relationship between income and whether the person played the lottery.
c. Reject H_0 if χ^2 is greater than 5.991.
d.

$$\chi^2 = \frac{(46-40.71)^2}{40.71} + \frac{(28-27.14)^2}{27.14} + \frac{(21-27.14)^2}{27.14} + \frac{(14-19.29)^2}{19.29} + \frac{(12-12.86)^2}{12.86} + \frac{(19-12.86)^2}{12.86}$$
$$= 6.544$$

e. Reject H_0. There is a relationship between income level and playing the lottery.